

# NAG Library Routine Document

## G08RBF

**Note:** before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

### 1 Purpose

G08RBF calculates the parameter estimates, score statistics and their variance-covariance matrices for the linear model using a likelihood based on the ranks of the observations when some of the observations may be right-censored.

### 2 Specification

```

SUBROUTINE G08RBF (NS, NV, NSUM, Y, IP, X, LDX, ICEN, GAMMA, NMAX, TOL,      &
                  PRVR, LDPRVR, IRANK, ZIN, ETA, VAPVEC, PAREST, WORK,      &
                  LWORK, IWA, IFAIL)
INTEGER          NS, NV(NS), NSUM, IP, LDX, ICEN(NSUM), NMAX, LDPRVR,      &
                  IRANK(NMAX), LWORK, IWA(4*NMAX), IFAIL
REAL (KIND=nag_wp) Y(NSUM), X(LDX,IP), GAMMA, TOL, PRVR(LDPRVR,IP),      &
                  ZIN(NMAX), ETA(NMAX), VAPVEC(NMAX*(NMAX+1)/2),          &
                  PAREST(4*IP+1), WORK(LWORK)

```

### 3 Description

Analysis of data can be made by replacing observations by their ranks. The analysis produces inference for the regression model where the location parameters of the observations,  $\theta_i$ , for  $i = 1, 2, \dots, n$ , are related by  $\theta = X\beta$ . Here  $X$  is an  $n$  by  $p$  matrix of explanatory variables and  $\beta$  is a vector of  $p$  unknown regression parameters. The observations are replaced by their ranks and an approximation, based on Taylor's series expansion, made to the rank marginal likelihood. For details of the approximation see Pettitt (1982).

An observation is said to be right-censored if we can only observe  $Y_j^*$  with  $Y_j^* \leq Y_j$ . We rank censored and uncensored observations as follows. Suppose we can observe  $Y_j$ , for  $j = 1, 2, \dots, n$ , directly but  $Y_j^*$ , for  $j = n+1, \dots, q$  and  $n \leq q$ , are censored on the right. We define the rank  $r_j$  of  $Y_j$ , for  $j = 1, 2, \dots, n$ , in the usual way;  $r_j$  equals  $i$  if and only if  $Y_j$  is the  $i$ th smallest amongst the  $Y_1, Y_2, \dots, Y_n$ . The right-censored  $Y_j^*$ , for  $j = n+1, n+2, \dots, q$ , has rank  $r_j$  if and only if  $Y_j^*$  lies in the interval  $[Y_{(r_j)}, Y_{(r_j+1)}]$ , with  $Y_0 = -\infty$ ,  $Y_{(n+1)} = +\infty$  and  $Y_{(1)} < \dots < Y_{(n)}$  the ordered  $Y_j$ , for  $j = 1, 2, \dots, n$ .

The distribution of the  $Y$  is assumed to be of the following form. Let  $F_L(y) = e^y/(1 + e^y)$ , the logistic distribution function, and consider the distribution function  $F_\gamma(y)$  defined by  $1 - F_\gamma = [1 - F_L(y)]^{1/\gamma}$ . This distribution function can be thought of as either the distribution function of the minimum,  $X_{1,\gamma}$ , of a random sample of size  $\gamma^{-1}$  from the logistic distribution, or as the  $F_\gamma(y - \log \gamma)$  being the distribution function of a random variable having the  $F$ -distribution with 2 and  $2\gamma^{-1}$  degrees of freedom. This family of generalized logistic distribution functions  $[F_\gamma(\cdot); 0 \leq \gamma < \infty]$  naturally links the symmetric logistic distribution ( $\gamma = 1$ ) with the skew extreme value distribution ( $\lim \gamma \rightarrow 0$ ) and with the limiting negative exponential distribution ( $\lim \gamma \rightarrow \infty$ ). For this family explicit results are available for right-censored data. See Pettitt (1983) for details.

Let  $l_R$  denote the logarithm of the rank marginal likelihood of the observations and define the  $q \times 1$  vector  $a$  by  $a = l'_R(\theta = 0)$ , and let the  $q$  by  $q$  diagonal matrix  $B$  and  $q$  by  $q$  symmetric matrix  $A$  be given by  $B - A = -l''_R(\theta = 0)$ . Then various statistics can be found from the analysis.

- The score statistic  $X^T a$ . This statistic is used to test the hypothesis  $H_0 : \beta = 0$  (see (e)).
- The estimated variance-covariance matrix of the score statistic in (a).

- (c) The estimate  $\hat{\beta}_R = MX^T a$ .
- (d) The estimated variance-covariance matrix  $M = (X^T(B - A)X)^{-1}$  of the estimate  $\hat{\beta}_R$ .
- (e) The  $\chi^2$  statistic  $Q = \hat{\beta}_R M^{-1} \hat{\beta}_R = a^T X (X^T(B - A)X)^{-1} X^T a$ , used to test  $H_0 : \beta = 0$ . Under  $H_0$ ,  $Q$  has an approximate  $\chi^2$ -distribution with  $p$  degrees of freedom.
- (f) The standard errors  $M_{ii}^{1/2}$  of the estimates given in (c).
- (g) Approximate  $z$ -statistics, i.e.,  $Z_i = \hat{\beta}_{R_i} / se(\hat{\beta}_{R_i})$  for testing  $H_0 : \beta_i = 0$ . For  $i = 1, 2, \dots, n$ ,  $Z_i$  has an approximate  $N(0, 1)$  distribution.

In many situations, more than one sample of observations will be available. In this case we assume the model,

$$h_k(Y_k) = X_k^T \beta + e_k, \quad k = 1, 2, \dots, \text{NS},$$

where NS is the number of samples. In an obvious manner,  $Y_k$  and  $X_k$  are the vector of observations and the design matrix for the  $k$ th sample respectively. Note that the arbitrary transformation  $h_k$  can be assumed different for each sample since observations are ranked within the sample.

The earlier analysis can be extended to give a combined estimate of  $\beta$  as  $\hat{\beta} = Dd$ , where

$$D^{-1} = \sum_{k=1}^{\text{NS}} X_k^T (B_k - A_k) X_k$$

and

$$d = \sum_{k=1}^{\text{NS}} X_k^T a_k,$$

with  $a_k$ ,  $B_k$  and  $A_k$  defined as  $a$ ,  $B$  and  $A$  above but for the  $k$ th sample.

The remaining statistics are calculated as for the one sample case.

## 4 References

Kalbfleisch J D and Prentice R L (1980) *The Statistical Analysis of Failure Time Data* Wiley

Pettitt A N (1982) Inference for the linear model using a likelihood based on ranks *J. Roy. Statist. Soc. Ser. B* **44** 234–243

Pettitt A N (1983) Approximate methods using ranks for regression with censored data *Biometrika* **70** 121–132

## 5 Arguments

1: NS – INTEGER *Input*

*On entry:* the number of samples.

*Constraint:* NS  $\geq$  1.

2: NV(NS) – INTEGER array *Input*

*On entry:* the number of observations in the  $i$ th sample, for  $i = 1, 2, \dots, \text{NS}$ .

*Constraint:* NV( $i$ )  $\geq$  1, for  $i = 1, 2, \dots, \text{NS}$ .

3: NSUM – INTEGER *Input*

*On entry:* the total number of observations.

*Constraint:* NSUM =  $\sum_{i=1}^{\text{NS}} \text{NV}(i)$ .

- 4: Y(NSUM) – REAL (KIND=nag\_wp) array Input  
*On entry:* the observations in each sample. Specifically,  $Y(\sum_{k=1}^{i-1} NV(k) + j)$  must contain the  $j$ th observation in the  $i$ th sample.
- 5: IP – INTEGER Input  
*On entry:* the number of parameters to be fitted.  
*Constraint:*  $IP \geq 1$ .
- 6: X(LDX, IP) – REAL (KIND=nag\_wp) array Input  
*On entry:* the design matrices for each sample. Specifically,  $X(\sum_{k=1}^{i-1} NV(k) + j, l)$  must contain the value of the  $l$ th explanatory variable for the  $j$ th observations in the  $i$ th sample.  
*Constraint:* X must not contain a column with all elements equal.
- 7: LDX – INTEGER Input  
*On entry:* the first dimension of the array X as declared in the (sub)program from which G08RBF is called.  
*Constraint:*  $LDX \geq NSUM$ .
- 8: ICEN(NSUM) – INTEGER array Input  
*On entry:* defines the censoring variable for the observations in Y.  
 $ICEN(i) = 0$   
     If  $Y(i)$  is uncensored.  
 $ICEN(i) = 1$   
     If  $Y(i)$  is censored.  
*Constraint:*  $ICEN(i) = 0$  or  $1$ , for  $i = 1, 2, \dots, NSUM$ .
- 9: GAMMA – REAL (KIND=nag\_wp) Input  
*On entry:* the value of the parameter defining the generalized logistic distribution. For  $GAMMA \leq 0.0001$ , the limiting extreme value distribution is assumed.  
*Constraint:*  $GAMMA \geq 0.0$ .
- 10: NMAX – INTEGER Input  
*On entry:* the value of the largest sample size.  
*Constraint:*  $NMAX = \max_{1 \leq i \leq NS} (NV(i))$  and  $NMAX > IP$ .
- 11: TOL – REAL (KIND=nag\_wp) Input  
*On entry:* the tolerance for judging whether two observations are tied. Thus, observations  $Y_i$  and  $Y_j$  are adjudged to be tied if  $|Y_i - Y_j| < TOL$ .  
*Constraint:*  $TOL > 0.0$ .
- 12: PRVR(LDPRVR, IP) – REAL (KIND=nag\_wp) array Output  
*On exit:* the variance-covariance matrices of the score statistics and the parameter estimates, the former being stored in the upper triangle and the latter in the lower triangle. Thus for  $1 \leq i \leq j \leq IP$ ,  $PRVR(i, j)$  contains an estimate of the covariance between the  $i$ th and  $j$ th score

statistics. For  $1 \leq j \leq i \leq IP - 1$ ,  $PRVR(i + 1, j)$  contains an estimate of the covariance between the  $i$ th and  $j$ th parameter estimates.

- 13: LDPRVR – INTEGER *Input*  
*On entry:* the first dimension of the array PRVR as declared in the (sub)program from which G08RBF is called.  
*Constraint:* LDPRVR  $\geq$  IP + 1.
- 14: IRANK(NMAX) – INTEGER array *Output*  
*On exit:* for the one sample case, IRANK contains the ranks of the observations.
- 15: ZIN(NMAX) – REAL (KIND=nag\_wp) array *Output*  
*On exit:* for the one sample case, ZIN contains the expected values of the function  $g(\cdot)$  of the order statistics.
- 16: ETA(NMAX) – REAL (KIND=nag\_wp) array *Output*  
*On exit:* for the one sample case, ETA contains the expected values of the function  $g(\cdot)$  of the order statistics.
- 17: VAPVEC(NMAX  $\times$  (NMAX + 1)/2) – REAL (KIND=nag\_wp) array *Output*  
*On exit:* for the one sample case, VAPVEC contains the upper triangle of the variance-covariance matrix of the function  $g(\cdot)$  of the order statistics stored column-wise.
- 18: PAREST( $4 \times IP + 1$ ) – REAL (KIND=nag\_wp) array *Output*  
*On exit:* the statistics calculated by the routine.  
 The first IP components of PAREST contain the score statistics.  
 The next IP elements contain the parameter estimates.  
 PAREST( $2 \times IP + 1$ ) contains the value of the  $\chi^2$  statistic.  
 The next IP elements of PAREST contain the standard errors of the parameter estimates.  
 Finally, the remaining IP elements of PAREST contain the  $z$ -statistics.
- 19: WORK(LWORK) – REAL (KIND=nag\_wp) array *Workspace*  
 20: LWORK – INTEGER *Input*  
*On entry:* the dimension of the array WORK as declared in the (sub)program from which G08RBF is called.  
*Constraint:* LWORK  $\geq$  NMAX  $\times$  (IP + 1).
- 21: IWA( $4 \times$  NMAX) – INTEGER array *Workspace*
- 22: IFAIL – INTEGER *Input/Output*  
*On entry:* IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this argument you should refer to Section 3.4 in How to Use the NAG Library and its Documentation for details.  
 For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this argument, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**  
*On exit:* IFAIL = 0 unless the routine detects an error or a warning has been flagged (see Section 6).

## 6 Error Indicators and Warnings

If on entry  $IFAIL = 0$  or  $-1$ , explanatory error messages are output on the current error message unit (as defined by  $X04AAF$ ).

Errors or warnings detected by the routine:

$IFAIL = 1$

On entry,  $NS < 1$ ,  
 or  $TOL \leq 0.0$ ,  
 or  $NMAX \leq IP$ ,  
 or  $LDPRVR < IP + 1$ ,  
 or  $LDX < NSUM$ ,  
 or  $NMAX \neq \max_{1 \leq i \leq NS}(NV(i))$ ,  
 or  $NV(i) \leq 0$  for some  $i$ ,  $i = 1, 2, \dots, NS$ ,  
 or  $NSUM \neq \sum_{i=1}^{NS} NV(i)$ ,  
 or  $IP < 1$ ,  
 or  $GAMMA < 0.0$ ,  
 or  $LWORK < NMAX \times (IP + 1)$ .

$IFAIL = 2$

On entry,  $ICEN(i) \neq 0$  or  $1$ , for some  $1 \leq i \leq NSUM$ .

$IFAIL = 3$

On entry, all the observations are adjudged to be tied. You are advised to check the value supplied for  $TOL$ .

$IFAIL = 4$

The matrix  $X^T(B - A)X$  is either ill-conditioned or not positive definite. This error should only occur with extreme rankings of the data.

$IFAIL = 5$

On entry, at least one column of the matrix  $X$  has all its elements equal.

$IFAIL = -99$

An unexpected error has been triggered by this routine. Please contact NAG.

See Section 3.9 in How to Use the NAG Library and its Documentation for further information.

$IFAIL = -399$

Your licence key may have expired or may not have been installed correctly.

See Section 3.8 in How to Use the NAG Library and its Documentation for further information.

$IFAIL = -999$

Dynamic memory allocation failed.

See Section 3.7 in How to Use the NAG Library and its Documentation for further information.

## 7 Accuracy

The computations are believed to be stable.

## 8 Parallelism and Performance

G08RBF is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

G08RBF makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this routine. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

## 9 Further Comments

The time taken by G08RBF depends on the number of samples, the total number of observations and the number of parameters fitted.

In extreme cases the parameter estimates for certain models can be infinite, although this is unlikely to occur in practice. See Pettitt (1982) for further details.

## 10 Example

This example fits a regression model to a single sample of 40 observations using just one explanatory variable.

### 10.1 Program Text

```

Program g08rbfe

!      G08RBF Example Program Text

!      Mark 26 Release. NAG Copyright 2016.

!      .. Use Statements ..
      Use nag_library, Only: g08rbf, nag_wp
!      .. Implicit None Statement ..
      Implicit None
!      .. Parameters ..
      Integer, Parameter          :: nin = 5, nout = 6
!      .. Local Scalars ..
      Real (Kind=nag_wp)         :: gamma, tol
      Integer                    :: i, ifail, ip, j, ldprvr, ldx, liwa, &
                                lparest, lvapvec, lwork, nmax, ns, &
                                nsum
!      .. Local Arrays ..
      Real (Kind=nag_wp), Allocatable :: eta(:), parest(:), prvr(:, :), &
                                vapvec(:), work(:), x(:, :), y(:), &
                                zin(:)
      Integer, Allocatable          :: icen(:), irank(:), iwa(:), nv(:)
!      .. Intrinsic Procedures ..
      Intrinsic                    :: maxval, sum
!      .. Executable Statements ..
      Write (nout,*) 'G08RBF Example Program Results'
      Write (nout,*)

!      Skip heading in data file
      Read (nin,*)

!      Read number of samples, number of parameters to be fitted,
!      distribution power parameter and tolerance criterion for ties.
      Read (nin,*) ns, ip, gamma, tol

      Allocate (nv(ns))

!      Read the number of observations in each sample

```

```

      Read (nin,*) nv(1:ns)

!      Calculate NSUM, NMAX and various array lengths
      nsum = sum(nv(1:ns))
      nmax = maxval(nv(1:ns))
      ldx = nsum
      ldprvr = ip + 1
      lvapvec = nmax*(nmax+1)/2
      lparest = 4*ip + 1
      lwork = nmax*(ip+1)
      liwa = 4*nmax

      Allocate (y(nsum),x(ldx,ip),icen(nsum),prvr(ldprvr,ip),irank(nmax),      &
               zin(nmax),eta(nmax),vapvec(lvapvec),parest(lparest),work(lwork),      &
               iwa(liwa))

!      Read in observations, design matrix and censoring variable
      Read (nin,*)(y(i),x(i,1:ip),icen(i),i=1,nsum)

!      Display input information
      Write (nout,99999) 'Number of samples =', ns
      Write (nout,99999) 'Number of parameters fitted =', ip
      Write (nout,99998) 'Distribution power parameter =', gamma
      Write (nout,99998) 'Tolerance for ties =', tol

      ifail = 0
      Call g08rbf(ns,nv,nsum,y,ip,x,ldx,icen,gamma,nmax,tol,prvr,ldprvr,irank, &
                zin,eta,vapvec,parest,work,lwork,iwa,ifail)

!      Display results
      Write (nout,*)
      Write (nout,*) 'Score statistic'
      Write (nout,99997) parest(1:ip)
      Write (nout,*)
      Write (nout,*) 'Covariance matrix of score statistic'
      Do j = 1, ip
         Write (nout,99997) prvr(1:j,j)
      End Do
      Write (nout,*)
      Write (nout,*) 'Parameter estimates'
      Write (nout,99997) parest((ip+1):(2*ip))
      Write (nout,*)
      Write (nout,*) 'Covariance matrix of parameter estimates'
      Do i = 1, ip
         Write (nout,99997) prvr(i+1,1:i)
      End Do
      Write (nout,*)
      Write (nout,99996) 'Chi-squared statistic =', parest(2*ip+1), ' with',      &
         ip, ' d.f.'
      Write (nout,*)
      Write (nout,*) 'Standard errors of estimates and'
      Write (nout,*) 'approximate z-statistics'
      Write (nout,99995)(parest(2*ip+1+i),parest(3*ip+1+i),i=1,ip)

99999 Format (1X,A,I2)
99998 Format (1X,A,F10.5)
99997 Format (1X,F9.3)
99996 Format (1X,A,F9.3,A,I2,A)
99995 Format (1X,F9.3,F14.3)
      End Program g08rbfe

```

## 10.2 Program Data

G08RBF Example Program Data

```

1 1 0.00001 0.00001
40
143.0 0.0 0 164.0 0.0 0 188.0 0.0 0 188.0 0.0 0 190.0 0.0 0
192.0 0.0 0 206.0 0.0 0 209.0 0.0 0 213.0 0.0 0 216.0 0.0 0
220.0 0.0 0 227.0 0.0 0 230.0 0.0 0 234.0 0.0 0 246.0 0.0 0

```

```
265.0 0.0 0 304.0 0.0 0 216.0 0.0 1 244.0 0.0 1 142.0 1.0 0
156.0 1.0 0 163.0 1.0 0 198.0 1.0 0 205.0 1.0 0 232.0 1.0 0
232.0 1.0 0 233.0 1.0 0 233.0 1.0 0 233.0 1.0 0 233.0 1.0 0
239.0 1.0 0 240.0 1.0 0 261.0 1.0 0 280.0 1.0 0 280.0 1.0 0
296.0 1.0 0 296.0 1.0 0 323.0 1.0 0 204.0 1.0 1 344.0 1.0 1
```

### 10.3 Program Results

G08RBF Example Program Results

```
Number of samples = 1
Number of parameters fitted = 1
Distribution power parameter = 0.00001
Tolerance for ties = 0.00001
```

```
Score statistic
4.584
```

```
Covariance matrix of score statistic
7.653
```

```
Parameter estimates
0.599
```

```
Covariance matrix of parameter estimates
0.131
```

```
Chi-squared statistic = 2.746 with 1 d.f.
```

```
Standard errors of estimates and
approximate z-statistics
0.361 1.657
```

---