

NAG Library Routine Document

F07HFF (DPBEQU)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

1 Purpose

F07HFF (DPBEQU) computes a diagonal scaling matrix S intended to equilibrate a real n by n symmetric positive definite band matrix A , with bandwidth $(2k_d + 1)$, and reduce its condition number.

2 Specification

```
SUBROUTINE F07HFF (UPLO, N, KD, AB, LDAB, S, SCOND, AMAX, INFO)
INTEGER          N, KD, LDAB, INFO
REAL (KIND=nag_wp) AB(LDAB,*), S(N), SCOND, AMAX
CHARACTER(1)    UPLO
```

The routine may be called by its LAPACK name *dpbequ*.

3 Description

F07HFF (DPBEQU) computes a diagonal scaling matrix S chosen so that

$$s_j = 1/\sqrt{a_{jj}}.$$

This means that the matrix B given by

$$B = SAS,$$

has diagonal elements equal to unity. This in turn means that the condition number of B , $\kappa_2(B)$, is within a factor n of the matrix of smallest possible condition number over all possible choices of diagonal scalings (see Corollary 7.6 of Higham (2002)).

4 References

Higham N J (2002) *Accuracy and Stability of Numerical Algorithms* (2nd Edition) SIAM, Philadelphia

5 Arguments

- 1: UPLO – CHARACTER(1) *Input*
On entry: indicates whether the upper or lower triangular part of A is stored in the array AB, as follows:
UPLO = 'U'
The upper triangle of A is stored.
UPLO = 'L'
The lower triangle of A is stored.
Constraint: UPLO = 'U' or 'L'.
- 2: N – INTEGER *Input*
On entry: n , the order of the matrix A .
Constraint: $N \geq 0$.

- 3: KD – INTEGER *Input*
On entry: k_d , the number of superdiagonals of the matrix A if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'.
Constraint: $KD \geq 0$.
- 4: AB(LDAB,*) – REAL (KIND=nag_wp) array *Input*
Note: the second dimension of the array AB must be at least $\max(1, N)$.
On entry: the upper or lower triangle of the symmetric positive definite band matrix A whose scaling factors are to be computed.
 The matrix is stored in rows 1 to $k_d + 1$, more precisely,
 if UPLO = 'U', the elements of the upper triangle of A within the band must be stored with element A_{ij} in $AB(k_d + 1 + i - j, j)$ for $\max(1, j - k_d) \leq i \leq j$;
 if UPLO = 'L', the elements of the lower triangle of A within the band must be stored with element A_{ij} in $AB(1 + i - j, j)$ for $j \leq i \leq \min(n, j + k_d)$.
 Only the elements of the array AB corresponding to the diagonal elements of A are referenced. (Row $(k_d + 1)$ of AB when UPLO = 'U', row 1 of AB when UPLO = 'L'.)
- 5: LDAB – INTEGER *Input*
On entry: the first dimension of the array AB as declared in the (sub)program from which F07HFF (DPBEQU) is called.
Constraint: $LDAB \geq KD + 1$.
- 6: S(N) – REAL (KIND=nag_wp) array *Output*
On exit: if INFO = 0, S contains the diagonal elements of the scaling matrix S .
- 7: SCOND – REAL (KIND=nag_wp) *Output*
On exit: if INFO = 0, SCOND contains the ratio of the smallest value of S to the largest value of S. If $SCOND \geq 0.1$ and AMAX is neither too large nor too small, it is not worth scaling by S .
- 8: AMAX – REAL (KIND=nag_wp) *Output*
On exit: $\max |a_{ij}|$. If AMAX is very close to overflow or underflow, the matrix A should be scaled.
- 9: INFO – INTEGER *Output*
On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

INFO < 0

If INFO = $-i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

The $\langle value \rangle$ th diagonal element of A is not positive (and hence A cannot be positive definite).

7 Accuracy

The computed scale factors will be close to the exact scale factors.

8 Parallelism and Performance

F07HFF (DPBEQU) is not threaded in any implementation.

9 Further Comments

The complex analogue of this routine is F07HTF (ZPBEQU).

10 Example

This example equilibrates the symmetric positive definite matrix A given by

$$A = \begin{pmatrix} 5.49 & 2.68 \times 10^{10} & 0 & 0 \\ 2.68 \times 10^{10} & 5.63 \times 10^{20} & -2.39 \times 10^{10} & 0 \\ 0 & -2.39 \times 10^{10} & 2.60 & -2.22 \\ 0 & 0 & -2.22 & 5.17 \end{pmatrix}.$$

Details of the scaling factors and the scaled matrix are output.

10.1 Program Text

```

Program f07hffe

!      F07HFF Example Program Text

!      Mark 26 Release. NAG Copyright 2016.

!      .. Use Statements ..
!      Use nag_library, Only: dpbequ, dscal, f06fcf, nag_wp, x02ajf, x02amf,      &
!                               x02bhf, x04cef

!      .. Implicit None Statement ..
!      Implicit None

!      .. Parameters ..
!      Real (Kind=nag_wp), Parameter      :: one = 1.0_nag_wp
!      Real (Kind=nag_wp), Parameter      :: thresh = 0.1_nag_wp
!      Integer, Parameter                  :: nin = 5, nout = 6
!      Character (1), Parameter            :: uplo = 'U'

!      .. Local Scalars ..
!      Real (Kind=nag_wp)                  :: amax, big, scond, small
!      Integer                              :: i, i0, il, ifail, ilen, info, j, kd, &
!                                           ldab, n

!      .. Local Arrays ..
!      Real (Kind=nag_wp), Allocatable     :: ab(:, :), s(:)

!      .. Intrinsic Procedures ..
!      Intrinsic                          :: max, min, real

!      .. Executable Statements ..
!      Write (nout,*) 'F07HFF Example Program Results'
!      Write (nout,*)
!      Flush (nout)

!      Skip heading in data file
!      Read (nin,*)
!      Read (nin,*) n, kd
!      ldab = kd + 1
!      Allocate (ab(ldab,n),s(n))

!      Read the upper or lower triangular part of the band matrix A
!      from data file

!      If (uplo=='U') Then
!         Do i = 1, n
!            Read (nin,*)(ab(kd+1+i-j,j),j=i,min(n,i+kd))
!         End Do
!      Else If (uplo=='L') Then
!         Do i = 1, n
!            Read (nin,*)(ab(1+i-j,j),j=max(1,i-kd),i)
!         End Do
!      End If

```

```

!      Print the matrix A

!      ifail: behaviour on error exit
!      =0 for hard exit, =1 for quiet-soft, =-1 for noisy-soft
      ifail = 0
      If (uplo=='U') Then
        Call x04cef(n,n,0,kd,ab,ldab,'Matrix A',ifail)
      Else If (uplo=='L') Then
        Call x04cef(n,n,kd,0,ab,ldab,'Matrix A',ifail)
      End If

      Write (nout,*)

!      Compute diagonal scaling factors
!      The NAG name equivalent of dpbequ is f07hff
      Call dpbequ(uplo,n,kd,ab,ldab,s,scond,amax,info)

      If (info>0) Then
        Write (nout,99999) 'Diagonal element', info, ' of A is non positive'
      Else

!      Print SCOND, AMAX and the scale factors

      Write (nout,99998) 'SCOND =', sconfd, ', AMAX =', amax
      Write (nout,*)
      Write (nout,*) 'Diagonal scaling factors'
      Write (nout,99997) s(1:n)
      Write (nout,*)
      Flush (nout)

!      Compute values close to underflow and overflow

      small = x02amf()/(x02ajf()*real(x02bhf(),kind=nag_wp))
      big = one/small
      If ((scond<thresh) .Or. (amax<small) .Or. (amax>big)) Then

!      Scale A
      If (uplo=='U') Then

!      The NAG name equivalent of dscal is f06edf
      Do j = 1, n
        i0 = max(1,j-kd)
        i1 = 1 + i0 - (j-kd)
        ilen = j - i0 + 1
        Call dscal(ilen,s(j),ab(i1,j),1)
        Call f06fcf(ilen,s(i0),1,ab(i1,j),1)
      End Do

      Else If (uplo=='L') Then
        Do j = 1, n
          i1 = 1
          ilen = min(n,j+kd) - j + 1
          Call dscal(ilen,s(j),ab(i1,j),1)
          Call f06fcf(ilen,s(j),1,ab(i1,j),1)
        End Do
      End If

!      Print the scaled matrix

      ifail = 0
      If (uplo=='U') Then
        Call x04cef(n,n,0,kd,ab,ldab,'Scaled matrix',ifail)
      Else If (uplo=='L') Then
        Call x04cef(n,n,kd,0,ab,ldab,'Scaled matrix',ifail)
      End If
    End If
  End If

```

```

99999 Format (1X,A,I4,A)
99998 Format (1X,2(A,1P,E8.1))
99997 Format ((1X,1P,7E11.1))
      End Program f07hffe

```

10.2 Program Data

```

F07HFF Example Program Data
  4 1                               :Values of N and KD
  5.49E+00  2.68E+10
           5.63E+20  -2.39E+10
                    2.60E+00  -2.22E+00
                              5.17E+00 :End of matrix A

```

10.3 Program Results

F07HFF Example Program Results

```

Matrix A
      1           2           3           4
  1  5.4900E+00  2.6800E+10
  2           5.6300E+20  -2.3900E+10
  3                    2.6000E+00  -2.2200E+00
  4                               5.1700E+00

```

SCOND = 6.8E-11, AMAX = 5.6E+20

```

Diagonal scaling factors
  4.3E-01  4.2E-11  6.2E-01  4.4E-01

```

```

Scaled matrix
      1           2           3           4
  1  1.0000  0.4821
  2           1.0000  -0.6247
  3                    1.0000  -0.6055
  4                               1.0000

```
