

NAG Library Routine Document

F07CAF (DGTSV)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

1 Purpose

F07CAF (DGTSV) computes the solution to a real system of linear equations

$$AX = B,$$

where A is an n by n tridiagonal matrix and X and B are n by r matrices.

2 Specification

```
SUBROUTINE F07CAF (N, NRHS, DL, D, DU, B, LDB, INFO)
  INTEGER          N, NRHS, LDB, INFO
  REAL (KIND=nag_wp) DL(*), D(*), DU(*), B(LDB,*)
```

The routine may be called by its LAPACK name *dgtsv*.

3 Description

F07CAF (DGTSV) uses Gaussian elimination with partial pivoting and row interchanges to solve the equations $AX = B$. The matrix A is factorized as $A = PLU$, where P is a permutation matrix, L is unit lower triangular with at most one nonzero subdiagonal element per column, and U is an upper triangular band matrix, with two superdiagonals.

Note that equations $A^T X = B$ may be solved by interchanging the order of the arguments DU and DL .

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia <http://www.netlib.org/lapack/lug>

5 Arguments

- 1: N – INTEGER *Input*
On entry: n , the number of linear equations, i.e., the order of the matrix A .
Constraint: $N \geq 0$.
- 2: NRHS – INTEGER *Input*
On entry: r , the number of right-hand sides, i.e., the number of columns of the matrix B .
Constraint: $NRHS \geq 0$.
- 3: DL(*) – REAL (KIND=nag_wp) array *Input/Output*
Note: the dimension of the array DL must be at least $\max(1, N - 1)$.
On entry: must contain the $(n - 1)$ subdiagonal elements of the matrix A .
On exit: if no constraints are violated, DL is overwritten by the $(n - 2)$ elements of the second superdiagonal of the upper triangular matrix U from the LU factorization of A , in $DL(1), DL(2), \dots, DL(n - 2)$.

- 4: D(*) – REAL (KIND=nag_wp) array Input/Output
Note: the dimension of the array D must be at least $\max(1, N)$.
On entry: must contain the n diagonal elements of the matrix A .
On exit: if no constraints are violated, D is overwritten by the n diagonal elements of the upper triangular matrix U from the LU factorization of A .
- 5: DU(*) – REAL (KIND=nag_wp) array Input/Output
Note: the dimension of the array DU must be at least $\max(1, N - 1)$.
On entry: must contain the $(n - 1)$ superdiagonal elements of the matrix A .
On exit: if no constraints are violated, DU is overwritten by the $(n - 1)$ elements of the first superdiagonal of U .
- 6: B(LDB, *) – REAL (KIND=nag_wp) array Input/Output
Note: the second dimension of the array B must be at least $\max(1, \text{NRHS})$.
 To solve the equations $Ax = b$, where b is a single right-hand side, B may be supplied as a one-dimensional array with length $\text{LDB} = \max(1, N)$.
On entry: the n by r right-hand side matrix B .
On exit: if $\text{INFO} = 0$, the n by r solution matrix X .
- 7: LDB – INTEGER Input
On entry: the first dimension of the array B as declared in the (sub)program from which F07CAF (DGTSV) is called.
Constraint: $\text{LDB} \geq \max(1, N)$.
- 8: INFO – INTEGER Output
On exit: $\text{INFO} = 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

INFO < 0

If $\text{INFO} = -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

Element $\langle \text{value} \rangle$ of the diagonal is exactly zero, and the solution has not been computed. The factorization has not been completed unless $N = \langle \text{value} \rangle$.

7 Accuracy

The computed solution for a single right-hand side, \hat{x} , satisfies an equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_1 = O(\epsilon)\|A\|_1$$

and ϵ is the *machine precision*. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x} - x\|_1}{\|x\|_1} \leq \kappa(A) \frac{\|E\|_1}{\|A\|_1},$$

where $\kappa(A) = \|A^{-1}\|_1 \|A\|_1$, the condition number of A with respect to the solution of the linear equations. See Section 4.4 of Anderson *et al.* (1999) for further details.

Alternatives to F07CAF (DGTSV), which return condition and error estimates are F04BCF and F07CBF (DGTSVX).

8 Parallelism and Performance

F07CAF (DGTSV) is not threaded in any implementation.

9 Further Comments

The total number of floating-point operations required to solve the equations $AX = B$ is proportional to nr .

The complex analogue of this routine is F07CNF (ZGTSV).

10 Example

This example solves the equations

$$Ax = b,$$

where A is the tridiagonal matrix

$$A = \begin{pmatrix} 3.0 & 2.1 & 0 & 0 & 0 \\ 3.4 & 2.3 & -1.0 & 0 & 0 \\ 0 & 3.6 & -5.0 & 1.9 & 0 \\ 0 & 0 & 7.0 & -0.9 & 8.0 \\ 0 & 0 & 0 & -6.0 & 7.1 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 2.7 \\ -0.5 \\ 2.6 \\ 0.6 \\ 2.7 \end{pmatrix}.$$

10.1 Program Text

```

Program f07cafe

!      F07CAF Example Program Text

!      Mark 26 Release. NAG Copyright 2016.

!      .. Use Statements ..
Use nag_library, Only: dgtsv, nag_wp
!      .. Implicit None Statement ..
Implicit None
!      .. Parameters ..
Integer, Parameter          :: nin = 5, nout = 6
!      .. Local Scalars ..
Integer                    :: info, n
!      .. Local Arrays ..
Real (Kind=nag_wp), Allocatable :: b(:), d(:), dl(:), du(:)
!      .. Executable Statements ..
Write (nout,*) 'F07CAF Example Program Results'
Write (nout,*)
!      Skip heading in data file
Read (nin,*)
Read (nin,*) n

Allocate (b(n),d(n),dl(n-1),du(n-1))

!      Read the tridiagonal matrix A and the right hand side B from
!      data file

Read (nin,*) du(1:n-1)
Read (nin,*) d(1:n)
Read (nin,*) dl(1:n-1)
Read (nin,*) b(1:n)

```

```

!      Solve the equations Ax = b for x

!      The NAG name equivalent of dgtsv is f07caf
      Call dgtsv(n,1,d1,d,du,b,n,info)

      If (info==0) Then

!         Print solution

         Write (nout,*) 'Solution'
         Write (nout,99999) b(1:n)

      Else
         Write (nout,99998) 'The (', info, ', ', info, ')',
           ' element of the factor U is zero'
      End If

99999 Format ((1X,7F11.4))
99998 Format (1X,A,I3,A,I3,A,A)
      End Program f07cafe

```

10.2 Program Data

```

F07CAF Example Program Data
5                               :Value of N
  2.1 -1.0  1.9  8.0
3.0  2.3 -5.0 -0.9  7.1
3.4  3.6  7.0 -6.0       :End of matrix A
2.7 -0.5  2.6  0.6  2.7 :End of vector B

```

10.3 Program Results

```

F07CAF Example Program Results

Solution
  -4.0000    7.0000    3.0000   -4.0000   -3.0000

```
