

## NAG Library Function Document

### nag\_surviv\_risk\_sets (g12zac)

#### 1 Purpose

nag\_surviv\_risk\_sets (g12zac) creates the risk sets associated with the Cox proportional hazards model for fixed covariates.

#### 2 Specification

```
#include <nag.h>
#include <nagg12.h>

void nag_surviv_risk_sets (Nag_OrderType order, Integer n, Integer m,
    Integer ns, const double z[], Integer pdz, const Integer isz[],
    Integer ip, const double t[], const Integer ic[], const Integer isi[],
    Integer *num, Integer ixs[], Integer *nxs, double x[], Integer mxn,
    Integer id[], Integer *nd, double tp[], Integer irs[], NagError *fail)
```

#### 3 Description

The Cox proportional hazards model (see Cox (1972)) relates the time to an event, usually death or failure, to a number of explanatory variables known as covariates. Some of the observations may be right-censored, that is, the exact time to failure is not known, only that it is greater than a known time.

Let  $t_i$ , for  $i = 1, 2, \dots, n$ , be the failure time or censored time for the  $i$ th observation with the vector of  $p$  covariates  $z_i$ . The covariance matrix  $Z$  is constructed so that it contains  $n$  rows with the  $i$ th row containing the  $p$  covariates  $z_i$ . It is assumed that censoring and failure mechanisms are independent. The hazard function,  $\lambda(t, z)$ , is the probability that an individual with covariates  $z$  fails at time  $t$  given that the individual survived up to time  $t$ . In the Cox proportional hazards model,  $\lambda(t, z)$  is of the form

$$\lambda(t, z) = \lambda_0(t) \exp(z^T \beta),$$

where  $\lambda_0$  is the base-line hazard function, an unspecified function of time, and  $\beta$  is a vector of unknown arguments. As  $\lambda_0$  is unknown, the arguments  $\beta$  are estimated using the conditional or marginal likelihood. This involves considering the covariate values of all subjects that are at risk at the time when a failure occurs. The probability that the subject that failed had their observed set of covariate values is computed.

The risk set at a failure time consists of those subjects that fail or are censored at that time and those who survive beyond that time. As risk sets are computed for every distinct failure time, it should be noted that the combined risk sets may be considerably larger than the original data. If the data can be considered as coming from different strata such that  $\lambda_0$  varies from strata to strata but  $\beta$  remains constant, then nag\_surviv\_risk\_sets (g12zac) will return a factor that indicates to which risk set/strata each member of the risk sets belongs rather than just to which risk set.

Given the risk sets the Cox proportional hazards model can then be fitted using a Poisson generalized linear model (nag\_glm\_poisson (g02gcc) with nag\_dummy\_vars (g04eac) to compute dummy variables) using Breslow's approximation for ties (see Breslow (1974)). This will give the same fit as nag\_surviv\_cox\_model (g12bac). If the exact treatment of ties in discrete time is required, as given by Cox (1972), then the model is fitted as a conditional logistic model using nag\_condl\_logistic (g11cac).

## 4 References

Breslow N E (1974) Covariate analysis of censored survival data *Biometrics* **30** 89–99

Cox D R (1972) Regression models in life tables (with discussion) *J. Roy. Statist. Soc. Ser. B* **34** 187–220

Gross A J and Clark V A (1975) *Survival Distributions: Reliability Applications in the Biomedical Sciences* Wiley

## 5 Arguments

1: **order** – Nag\_OrderType *Input*

*On entry:* the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag\_RowMajor. See Section 2.3.1.3 in How to Use the NAG Library and its Documentation for a more detailed explanation of the use of this argument.

*Constraint:* **order** = Nag\_RowMajor or Nag\_ColMajor.

2: **n** – Integer *Input*

*On entry:* *n*, the number of data points.

*Constraint:*  $n \geq 2$ .

3: **m** – Integer *Input*

*On entry:* the number of covariates in array **z**.

*Constraint:*  $m \geq 1$ .

4: **ns** – Integer *Input*

*On entry:* the number of strata. If **ns** > 0 then the stratum for each observation must be supplied in **isi**.

*Constraint:*  $ns \geq 0$ .

5: **z**[*dim*] – const double *Input*

**Note:** the dimension, *dim*, of the array **z** must be at least

$\max(1, \mathbf{pdz} \times \mathbf{m})$  when **order** = Nag\_ColMajor;  
 $\max(1, \mathbf{n} \times \mathbf{pdz})$  when **order** = Nag\_RowMajor.

The (*i*, *j*)th element of the matrix *Z* is stored in

$\mathbf{z}[(j-1) \times \mathbf{pdz} + i - 1]$  when **order** = Nag\_ColMajor;  
 $\mathbf{z}[(i-1) \times \mathbf{pdz} + j - 1]$  when **order** = Nag\_RowMajor.

*On entry:* must contain the *n* covariates in column or row major order.

6: **pdz** – Integer *Input*

*On entry:* the stride separating row or column elements (depending on the value of **order**) in the array **z**.

*Constraints:*

if **order** = Nag\_ColMajor,  $\mathbf{pdz} \geq \mathbf{n}$ ;  
 if **order** = Nag\_RowMajor,  $\mathbf{pdz} \geq \mathbf{m}$ .

- 7: **isz[m]** – const Integer *Input*  
*On entry:* indicates which subset of covariates are to be included in the model.  
**isz[j – 1] ≥ 1**  
The *j*th covariate is included in the model.  
**isz[j – 1] = 0**  
The *j*th covariate is excluded from the model and not referenced.  
*Constraint:* **isz[j – 1] ≥ 0** and at least one value must be nonzero.
- 8: **ip** – Integer *Input*  
*On entry:* *p*, the number of covariates included in the model as indicated by **isz**.  
*Constraint:* **ip** = the number of nonzero values of **isz**.
- 9: **t[n]** – const double *Input*  
*On entry:* the vector of *n* failure censoring times.
- 10: **ic[n]** – const Integer *Input*  
*On entry:* the status of the individual at time *t* given in **t**.  
**ic[i – 1] = 0**  
Indicates that the *i*th individual has failed at time **t[i – 1]**.  
**ic[i – 1] = 1**  
Indicates that the *i*th individual has been censored at time **t[i – 1]**.  
*Constraint:* **ic[i – 1] = 0** or **1**, for *i* = 1, 2, . . . , **n**.
- 11: **isi[dim]** – const Integer *Input*  
**Note:** the dimension, *dim*, of the array **isi** must be at least  
**n** when **ns** > 0;  
1 otherwise.  
*On entry:* if **ns** > 0, the stratum indicators which also allow data points to be excluded from the analysis.  
If **ns** = 0, **isi** is not referenced.  
**isi[i] = k**  
Indicates that the *i*th data point is in the *k*th stratum, where *k* = 1, 2, . . . , **ns**.  
**isi[i] = 0**  
Indicates that the *i*th data point is omitted from the analysis.  
*Constraint:* if **ns** > 0,  $0 \leq \mathbf{isi}[i] \leq \mathbf{ns}$ , for *i* = 0, 1, . . . , **n** – 1.
- 12: **num** – Integer \* *Output*  
*On exit:* the number of values in the combined risk sets.
- 13: **ixs[mxn]** – Integer *Output*  
*On exit:* the factor giving the risk sets/strata for the data in **x** and **id**.  
If **ns** = 0 or 1, **ixs[i – 1] = l** for members of the *l*th risk set.  
If **ns** > 1, **ixs[i – 1] = (j – 1) × nd + l** for the observations in the *l*th risk set for the *j*th strata.
- 14: **nxs** – Integer \* *Output*  
*On exit:* the number of levels for the risk sets/strata factor given in **ixs**.

- 15: **x**[**mxn** × **ip**] – double *Output*  
**Note:** the  $(i, j)$ th element of the matrix  $X$  is stored in  
 $\mathbf{x}[(j - 1) \times \mathbf{mxn} + i - 1]$  when **order** = Nag\_ColMajor;  
 $\mathbf{x}[(i - 1) \times \mathbf{ip} + j - 1]$  when **order** = Nag\_RowMajor.  
*On exit:* the first **num** rows contain the values of the covariates for the members of the risk sets.
- 16: **mxn** – Integer *Input*  
*On entry:* the first dimension of the array **x** and the dimension of the arrays **ixs** and **id**.  
**Constraint:** **mxn** must be sufficiently large for the arrays to contain the expanded risk sets. The size will depend on the pattern of failures times and censored times. The minimum value will be returned in **num** unless the function exits with **fail.code** = NE\_INT.
- 17: **id**[**mxn**] – Integer *Output*  
*On exit:* indicates if the member of the risk set given in **x** failed.  
**id**[ $i - 1$ ] = 1 if the member of the risk set failed at the time defining the risk set and **id**[ $i - 1$ ] = 0 otherwise.
- 18: **nd** – Integer \* *Output*  
*On exit:* the number of distinct failure times, i.e., the number of risk sets.
- 19: **tp**[**n**] – double *Output*  
*On exit:* **tp**[ $i - 1$ ] contains the  $i$ th distinct failure time, for  $i = 1, 2, \dots, \mathbf{nd}$ .
- 20: **irs**[**n**] – Integer *Output*  
*On exit:* indicates rows in **x** and elements in **ixs** and **id** corresponding to the risk sets. The first risk set corresponding to failure time **tp**[0] is given by rows 1 to **irs**[0]. The  $l$ th risk set is given by rows **irs**[ $l - 2$ ] + 1 to **irs**[ $l - 1$ ], for  $l = 1, 2, \dots, \mathbf{nd}$ .
- 21: **fail** – NagError \* *Input/Output*  
The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

### NE\_BAD\_PARAM

On entry, argument  $\langle value \rangle$  had an illegal value.

### NE\_INT

On entry, element  $\langle value \rangle$  of **ic** is not equal to 0 or 1.

On entry, element  $\langle value \rangle$  of **isi** is not valid.

On entry, element  $\langle value \rangle$  of **isz** < 0.

On entry, **m** =  $\langle value \rangle$ .

Constraint: **m** ≥ 1.

On entry, **n** =  $\langle value \rangle$ .

Constraint: **n**  $\geq 2$ .

On entry, **ns** =  $\langle value \rangle$ .

Constraint: **ns**  $\geq 0$ .

On entry, **pdz** =  $\langle value \rangle$ .

Constraint: **pdz**  $> 0$ .

## NE\_INT\_2

On entry, **pdz** =  $\langle value \rangle$  and **m** =  $\langle value \rangle$ .

Constraint: **pdz**  $\geq m$ .

## NE\_INT\_ARRAY\_ELEM\_CONS

**mxn** is too small: min value =  $\langle value \rangle$ .

On entry, there are not **ip** values of **isz**  $> 0$ .

## NE\_INTERNAL\_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.

See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

## NE\_NO\_LICENCE

Your licence key may have expired or may not have been installed correctly.

See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

## 7 Accuracy

Not applicable.

## 8 Parallelism and Performance

nag\_surviv\_risk\_sets (g12zac) is not threaded in any implementation.

## 9 Further Comments

When there are strata present, i.e., **ns**  $> 1$ , not all the **nxs** groups may be present.

## 10 Example

The data are the remission times for two groups of leukemia patients (see page 242 of Gross and Clark (1975)). A dummy variable indicates which group they come from. The risk sets are computed using nag\_surviv\_risk\_sets (g12zac) and the Cox's proportional hazard model is fitted using nag\_cond1\_logistic (g11cac).

### 10.1 Program Text

```
/* nag_surviv_risk_sets (g12zac) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */
```

```

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg11.h>
#include <nagg12.h>

int main(void)
{
    /* Scalars */
    double dev, tol;
    Integer exit_status, i, ip, iprint, j, lisi, m,
        maxit, mxn, n, nd, ns, num, nxs, pdx, pdz;
    NagError fail;
    Nag_OrderType order;

    /* Arrays */
    double *b = 0, *cov = 0, *sc = 0, *se = 0, *t = 0, *tp = 0, *x = 0, *z = 0;
    Integer *ic = 0, *id = 0, *irs = 0, *isi = 0, *isz = 0, *ixs = 0,
        *nca = 0, *nct = 0;

#ifdef NAG_COLUMN_MAJOR
#define Z(I, J) z[(J-1)*pdz + I - 1]
    order = Nag_ColMajor;
#else
#define Z(I, J) z[(I-1)*pdz + J - 1]
    order = Nag_RowMajor;
#endif

    INIT_FAIL(fail);

    exit_status = 0;

    printf("nag_surviv_risk_sets (g12zac) Example Program Results\n");

    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif

#ifdef _WIN32
    scanf_s("%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT
        "%*[\n] ", &n, &m, &ns, &maxit, &iprint);
#else
    scanf("%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT
        "%*[\n] ", &n, &m, &ns, &maxit, &iprint);
#endif

    /* Allocate arrays t, z, ic and isi */
    if (ns > 0)
        lisi = n;
    else
        lisi = 1;
    if (!(t = NAG_ALLOC(n, double)) ||
        !(z = NAG_ALLOC(n * n, double)) ||
        !(ic = NAG_ALLOC(n, Integer)) ||
        !(isi = NAG_ALLOC(lisi, Integer)) || !(isz = NAG_ALLOC(m, Integer)))
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }

    if (order == Nag_ColMajor) {
        pdz = n;
    }
    else {
        pdz = m;
    }
}

```

```

    if (ns > 0) {
        for (i = 1; i <= n; ++i) {
#ifdef _WIN32
            scanf_s("%lf", &t[i - 1]);
#else
            scanf("%lf", &t[i - 1]);
#endif
            for (j = 1; j <= m; ++j)
#ifdef _WIN32
                scanf_s("%lf", &Z(i, j));
#else
                scanf("%lf", &Z(i, j));
#endif
#ifdef _WIN32
            scanf_s("%" NAG_IFMT "%" NAG_IFMT "%*[\n] ", &ic[i - 1], &isi[i - 1]);
#else
            scanf("%" NAG_IFMT "%" NAG_IFMT "%*[\n] ", &ic[i - 1], &isi[i - 1]);
#endif
        }
    }
    else {
        for (i = 1; i <= n; ++i) {
#ifdef _WIN32
            scanf_s("%lf", &t[i - 1]);
#else
            scanf("%lf", &t[i - 1]);
#endif
            for (j = 1; j <= m; ++j)
#ifdef _WIN32
                scanf_s("%lf", &Z(i, j));
#else
                scanf("%lf", &Z(i, j));
#endif
#ifdef _WIN32
            scanf_s("%" NAG_IFMT "%*[\n] ", &ic[i - 1]);
#else
            scanf("%" NAG_IFMT "%*[\n] ", &ic[i - 1]);
#endif
        }
    }

    for (i = 1; i <= m; ++i)
#ifdef _WIN32
        scanf_s("%" NAG_IFMT "", &isz[i - 1]);
#else
        scanf("%" NAG_IFMT "", &isz[i - 1]);
#endif
#ifdef _WIN32
        scanf_s("%" NAG_IFMT "%*[\n] ", &ip);
#else
        scanf("%" NAG_IFMT "%*[\n] ", &ip);
#endif

    /* Allocate other arrays for nag_surviv_risk_sets (g12zac) */
    mxn = 1000;

    if (order == Nag_ColMajor) {
        pdx = mxn;
    }
    else {
        pdx = ip;
    }

    if (!(cov = NAG_ALLOC(ip * (ip + 1) / 2, double)) ||
        !(sc = NAG_ALLOC(ip, double)) ||
        !(se = NAG_ALLOC(ip, double)) ||
        !(tp = NAG_ALLOC(n, double)) ||
        !(x = NAG_ALLOC(mxn * ip, double)) ||
        !(id = NAG_ALLOC(mxn, Integer)) ||
        !(irs = NAG_ALLOC(n, Integer)) || !(ixs = NAG_ALLOC(mxn, Integer)))
    {

```

```

    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
}

/* nag_surviv_risk_sets (g12zac).
 * Creates the risk sets associated with the Cox
 * proportional hazards model for fixed covariates
 */
nag_surviv_risk_sets(order, n, m, ns, z, pdz, isz, ip, t, ic, isi, &num,
                    ix, &nxs, x, mxn, id, &nd, tp, irs, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_surviv_risk_sets (g12zac).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

/* Allocate arrays for nag_condl_logistic (g11cac) */
if (!(b = NAG_ALLOC(ip, double)) ||
    !(nca = NAG_ALLOC(nxs, Integer)) || !(nct = NAG_ALLOC(nxs, Integer)))
{
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
}

    for (i = 1; i <= ip; ++i)
#ifdef _WIN32
        scanf_s("%lf", &b[i - 1]);
#else
        scanf("%lf", &b[i - 1]);
#endif
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif

    tol = 1e-5;
/* nag_condl_logistic (g11cac).
 * Returns parameter estimates for the conditional analysis
 * of stratified data
 */
nag_condl_logistic(order, num, ip, nxs, x, pdx, isz, ip, id, ix, &dev, b,
                  se, sc, cov, nca, nct, tol, maxit, iprint, 0, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_condl_logistic (g11cac).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

printf("\n");
printf(" Parameter          Estimate          Standard Error\n");
printf("\n");
for (i = 1; i <= ip; ++i)
    printf("%5" NAG_IFMT "          %8.4f          %8.4f          \n",
          i, b[i - 1], se[i - 1]);

END:
    NAG_FREE(b);
    NAG_FREE(cov);
    NAG_FREE(sc);
    NAG_FREE(se);
    NAG_FREE(t);
    NAG_FREE(tp);
    NAG_FREE(x);
    NAG_FREE(z);
    NAG_FREE(ic);
    NAG_FREE(id);
    NAG_FREE(irs);
    NAG_FREE(isi);

```

```

NAG_FREE(isz);
NAG_FREE(ixs);
NAG_FREE(nca);
NAG_FREE(nct);

return exit_status;
}

```

## 10.2 Program Data

nag\_surviv\_risk\_sets (g12zac) Example Program Data

42 1 0 20 0

```

1 0 0
1 0 0
2 0 0
2 0 0
3 0 0
4 0 0
4 0 0
5 0 0
5 0 0
8 0 0
8 0 0
8 0 0
8 0 0
11 0 0
11 0 0
12 0 0
12 0 0
15 0 0
17 0 0
22 0 0
23 0 0
6 1 0
6 1 0
6 1 0
7 1 0
10 1 0
13 1 0
16 1 0
22 1 0
23 1 0
6 1 1
9 1 1
10 1 1
11 1 1
17 1 1
19 1 1
20 1 1
25 1 1
32 1 1
32 1 1
34 1 1
35 1 1

```

1 1

0.0 0.0

## 10.3 Program Results

nag\_surviv\_risk\_sets (g12zac) Example Program Results

Parameter	Estimate	Standard Error
1	1.6282	0.4331

---