

NAG Library Function Document

nag_zgetrs (f07asc)

1 Purpose

nag_zgetrs (f07asc) solves a complex system of linear equations with multiple right-hand sides,

$$AX = B, \quad A^T X = B \quad \text{or} \quad A^H X = B,$$

where A has been factorized by nag_zgetrf (f07arc).

2 Specification

```
#include <nag.h>
#include <nagf07.h>

void nag_zgetrs (Nag_OrderType order, Nag_TransType trans, Integer n,
                Integer nrhs, const Complex a[], Integer pda, const Integer ipiv[],
                Complex b[], Integer pdb, NagError *fail)
```

3 Description

nag_zgetrs (f07asc) is used to solve a complex system of linear equations $AX = B$, $A^T X = B$ or $A^H X = B$, the function must be preceded by a call to nag_zgetrf (f07arc) which computes the LU factorization of A as $A = PLU$. The solution is computed by forward and backward substitution.

If **trans** = Nag_NoTrans, the solution is computed by solving $PLY = B$ and then $UX = Y$.

If **trans** = Nag_Trans, the solution is computed by solving $U^T Y = B$ and then $L^T P^T X = Y$.

If **trans** = Nag_ConjTrans, the solution is computed by solving $U^H Y = B$ and then $L^H P^T X = Y$.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Arguments

1: **order** – Nag_OrderType *Input*

On entry: the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag_RowMajor. See Section 2.3.1.3 in How to Use the NAG Library and its Documentation for a more detailed explanation of the use of this argument.

Constraint: **order** = Nag_RowMajor or Nag_ColMajor.

2: **trans** – Nag_TransType *Input*

On entry: indicates the form of the equations.

trans = Nag_NoTrans
 $AX = B$ is solved for X .

trans = Nag_Trans
 $A^T X = B$ is solved for X .

trans = Nag_ConjTrans
 $A^H X = B$ is solved for X .

Constraint: **trans** = Nag_NoTrans, Nag_Trans or Nag_ConjTrans.

- 3: **n** – Integer *Input*
On entry: n , the order of the matrix A .
Constraint: $n \geq 0$.
- 4: **nrhs** – Integer *Input*
On entry: r , the number of right-hand sides.
Constraint: **nrhs** ≥ 0 .
- 5: **a**[*dim*] – const Complex *Input*
Note: the dimension, *dim*, of the array **a** must be at least $\max(1, \mathbf{pda} \times \mathbf{n})$.
The (i, j) th element of the matrix A is stored in
 $\mathbf{a}[(j-1) \times \mathbf{pda} + i - 1]$ when **order** = Nag_ColMajor;
 $\mathbf{a}[(i-1) \times \mathbf{pda} + j - 1]$ when **order** = Nag_RowMajor.
On entry: the LU factorization of A , as returned by nag_zgetrf (f07arc).
- 6: **pda** – Integer *Input*
On entry: the stride separating row or column elements (depending on the value of **order**) in the array **a**.
Constraint: **pda** $\geq \max(1, \mathbf{n})$.
- 7: **ipiv**[*dim*] – const Integer *Input*
Note: the dimension, *dim*, of the array **ipiv** must be at least $\max(1, \mathbf{n})$.
On entry: the pivot indices, as returned by nag_zgetrf (f07arc).
- 8: **b**[*dim*] – Complex *Input/Output*
Note: the dimension, *dim*, of the array **b** must be at least
 $\max(1, \mathbf{pdb} \times \mathbf{nrhs})$ when **order** = Nag_ColMajor;
 $\max(1, \mathbf{n} \times \mathbf{pdb})$ when **order** = Nag_RowMajor.
The (i, j) th element of the matrix B is stored in
 $\mathbf{b}[(j-1) \times \mathbf{pdb} + i - 1]$ when **order** = Nag_ColMajor;
 $\mathbf{b}[(i-1) \times \mathbf{pdb} + j - 1]$ when **order** = Nag_RowMajor.
On entry: the n by r right-hand side matrix B .
On exit: the n by r solution matrix X .
- 9: **pdb** – Integer *Input*
On entry: the stride separating row or column elements (depending on the value of **order**) in the array **b**.
Constraints:
if **order** = Nag_ColMajor, **pdb** $\geq \max(1, \mathbf{n})$;
if **order** = Nag_RowMajor, **pdb** $\geq \max(1, \mathbf{nrhs})$.

10: **fail** – NagError **Input/Output*

The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.

See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

NE_BAD_PARAM

On entry, argument $\langle value \rangle$ had an illegal value.

NE_INT

On entry, $\mathbf{n} = \langle value \rangle$.

Constraint: $\mathbf{n} \geq 0$.

On entry, $\mathbf{nrhs} = \langle value \rangle$.

Constraint: $\mathbf{nrhs} \geq 0$.

On entry, $\mathbf{pda} = \langle value \rangle$.

Constraint: $\mathbf{pda} > 0$.

On entry, $\mathbf{pdb} = \langle value \rangle$.

Constraint: $\mathbf{pdb} > 0$.

NE_INT_2

On entry, $\mathbf{pda} = \langle value \rangle$ and $\mathbf{n} = \langle value \rangle$.

Constraint: $\mathbf{pda} \geq \max(1, \mathbf{n})$.

On entry, $\mathbf{pdb} = \langle value \rangle$ and $\mathbf{n} = \langle value \rangle$.

Constraint: $\mathbf{pdb} \geq \max(1, \mathbf{n})$.

On entry, $\mathbf{pdb} = \langle value \rangle$ and $\mathbf{nrhs} = \langle value \rangle$.

Constraint: $\mathbf{pdb} \geq \max(1, \mathbf{nrhs})$.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.

See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.

See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

7 Accuracy

For each right-hand side vector b , the computed solution x is the exact solution of a perturbed system of equations $(A + E)x = b$, where

$$|E| \leq c(n)\epsilon P|L||U|,$$

$c(n)$ is a modest linear function of n , and ϵ is the *machine precision*.

If \hat{x} is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x - \hat{x}\|_{\infty}}{\|x\|_{\infty}} \leq c(n) \text{cond}(A, x)\epsilon$$

where $\text{cond}(A, x) = \frac{\| |A^{-1}| |A| |x| \|_{\infty}}{\|x\|_{\infty}} \leq \text{cond}(A) = \frac{\| |A^{-1}| |A| \|_{\infty}}{\|A\|_{\infty}} \leq \kappa_{\infty}(A)$.

Note that $\text{cond}(A, x)$ can be much smaller than $\text{cond}(A)$, and $\text{cond}(A^H)$ (which is the same as $\text{cond}(A^T)$) can be much larger (or smaller) than $\text{cond}(A)$.

Forward and backward error bounds can be computed by calling `nag_zgerfs` (f07avc), and an estimate for $\kappa_{\infty}(A)$ can be obtained by calling `nag_zgecon` (f07auc) with `norm = Nag_InfNorm`.

8 Parallelism and Performance

`nag_zgetrs` (f07asc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

`nag_zgetrs` (f07asc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the x06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

The total number of real floating-point operations is approximately $8n^2r$.

This function may be followed by a call to `nag_zgerfs` (f07avc) to refine the solution and return an error estimate.

The real analogue of this function is `nag_dgetrs` (f07aec).

10 Example

This example solves the system of equations $AX = B$, where

$$A = \begin{pmatrix} -1.34 + 2.55i & 0.28 + 3.17i & -6.39 - 2.20i & 0.72 - 0.92i \\ -0.17 - 1.41i & 3.31 - 0.15i & -0.15 + 1.34i & 1.29 + 1.38i \\ -3.29 - 2.39i & -1.91 + 4.42i & -0.14 - 1.35i & 1.72 + 1.35i \\ 2.41 + 0.39i & -0.56 + 1.47i & -0.83 - 0.69i & -1.96 + 0.67i \end{pmatrix}$$

and

$$B = \begin{pmatrix} 26.26 + 51.78i & 31.32 - 6.70i \\ 6.43 - 8.68i & 15.86 - 1.42i \\ -5.75 + 25.31i & -2.15 + 30.19i \\ 1.16 + 2.57i & -2.56 + 7.55i \end{pmatrix}.$$

Here A is nonsymmetric and must first be factorized by `nag_zgetrf` (f07arc).

10.1 Program Text

```
/* nag_zgetrs (f07asc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */
#include <stdio.h>
```

```

#include <nag.h>
#include <nag_stdlib.h>
#include <nagf07.h>
#include <nagx04.h>

int main(void)
{
    /* Scalars */
    Integer i, ipiv_len, j, n, nrhs, pda, pdb;
    Integer exit_status = 0;
    NagError fail;
    Nag_OrderType order;
    /* Arrays */
    Complex *a = 0, *b = 0;
    Integer *ipiv = 0;
#ifdef NAG_LOAD_FP
    /* The following line is needed to force the Microsoft linker
       to load floating point support */
    float force_loading_of_ms_float_support = 0;
#endif /* NAG_LOAD_FP */

#ifdef NAG_COLUMN_MAJOR
#define A(I, J) a[(J-1)*pda + I - 1]
#define B(I, J) b[(J-1)*pdb + I - 1]
    order = Nag_ColMajor;
#else
#define A(I, J) a[(I-1)*pda + J - 1]
#define B(I, J) b[(I-1)*pdb + J - 1]
    order = Nag_RowMajor;
#endif

    INIT_FAIL(fail);

    printf("nag_zgetrs (f07asc) Example Program Results\n\n");
    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif
#ifdef _WIN32
    scanf_s("%" NAG_IFMT "%" NAG_IFMT "%*[\n] ", &n, &nrhs);
#else
    scanf("%" NAG_IFMT "%" NAG_IFMT "%*[\n] ", &n, &nrhs);
#endif
#ifdef NAG_COLUMN_MAJOR
    pda = n;
    pdb = n;
#else
    pda = n;
    pdb = nrhs;
#endif
    ipiv_len = n;

    /* Allocate memory */
    if (!(a = NAG_ALLOC(n * n, Complex)) ||
        !(b = NAG_ALLOC(n * nrhs, Complex)) ||
        !(ipiv = NAG_ALLOC(ipiv_len, Integer)))
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }

    /* Read A and B from data file */
    for (i = 1; i <= n; ++i) {
        for (j = 1; j <= n; ++j) {
#ifdef _WIN32
            scanf_s(" ( %lf , %lf )", &A(i, j).re, &A(i, j).im);
#else
            scanf(" ( %lf , %lf )", &A(i, j).re, &A(i, j).im);
#endif
        }
    }
}

```

```

#endif
    }
}
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif
    for (i = 1; i <= n; ++i) {
        for (j = 1; j <= nrhs; ++j)
#ifdef _WIN32
            scanf_s(" ( %lf , %lf )", &B(i, j).re, &B(i, j).im);
#else
            scanf(" ( %lf , %lf )", &B(i, j).re, &B(i, j).im);
#endif
    }
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif

    /* Factorize A */
    /* nag_zgetrf (f07arc).
     * LU factorization of complex m by n matrix
     */
    nag_zgetrf(order, n, n, a, pda, ipiv, &fail);
    if (fail.code != NE_NOERROR) {
        printf("Error from nag_zgetrf (f07arc).\n%s\n", fail.message);
        exit_status = 1;
        goto END;
    }

    /* Compute solution */
    /* nag_zgetrs (f07asc).
     * Solution of complex system of linear equations, multiple
     * right-hand sides, matrix already factorized by nag_zgetrf
     * (f07arc)
     */
    nag_zgetrs(order, Nag_NoTrans, n, nrhs, a, pda, ipiv, b, pdb, &fail);
    if (fail.code != NE_NOERROR) {
        printf("Error from nag_zgetrs (f07asc).\n%s\n", fail.message);
        exit_status = 1;
        goto END;
    }

    /* Print solution */
    /* nag_gen_complx_mat_print_comp (x04dbc).
     * Print complex general matrix (comprehensive)
     */
    fflush(stdout);
    nag_gen_complx_mat_print_comp(order, Nag_GeneralMatrix, Nag_NonUnitDiag, n,
                                  nrhs, b, pdb, Nag_BracketForm, "%7.4f",
                                  "Solution(s)", Nag_IntegerLabels, 0,
                                  Nag_IntegerLabels, 0, 80, 0, 0, &fail);
    if (fail.code != NE_NOERROR) {
        printf("Error from nag_gen_complx_mat_print_comp (x04dbc).\n%s\n",
              fail.message);
        exit_status = 1;
        goto END;
    }
}
END:
    NAG_FREE(a);
    NAG_FREE(b);
    NAG_FREE(ipiv);
    return exit_status;
}

```

10.2 Program Data

```
nag_zgetrs (f07asc) Example Program Data
  4  2                                     :Values of N and NRHS
 (-1.34, 2.55) ( 0.28, 3.17) (-6.39,-2.20) ( 0.72,-0.92)
 (-0.17,-1.41) ( 3.31,-0.15) (-0.15, 1.34) ( 1.29, 1.38)
 (-3.29,-2.39) (-1.91, 4.42) (-0.14,-1.35) ( 1.72, 1.35)
 ( 2.41, 0.39) (-0.56, 1.47) (-0.83,-0.69) (-1.96, 0.67) :End of matrix A
 (26.26, 51.78) (31.32, -6.70)
 ( 6.43, -8.68) (15.86, -1.42)
 (-5.75, 25.31) (-2.15, 30.19)
 ( 1.16, 2.57) (-2.56, 7.55)                                     :End of matrix B
```

10.3 Program Results

```
nag_zgetrs (f07asc) Example Program Results
```

```
Solution(s)
           1           2
 1 ( 1.0000, 1.0000) (-1.0000,-2.0000)
 2 ( 2.0000,-3.0000) ( 5.0000, 1.0000)
 3 (-4.0000,-5.0000) (-3.0000, 4.0000)
 4 ( 0.0000, 6.0000) ( 2.0000,-3.0000)
```
