NAG Library Function Document

nag_asian_geom_greeks (s30sbc)

1 Purpose

nag_asian_geom_greeks (s30sbc) computes the Asian geometric continuous average-rate option price together with its sensitivities (Greeks).

2 Specification

```
#include <nag.h>
#include <nags.h>
```

```
void nag_asian_geom_greeks (Nag_OrderType order, Nag_CallPut option,
     Integer m, Integer n, const double x[], double s, const double t[],
     double sigma, double r, double b, double p[], double delta[],
     double gamma[], double vega[], double theta[], double rho[],
     double crho[], double vanna[], double charm[], double speed[],
     double colour[], double zomma[], double vomma[], NagError *fail)
```

3 Description

nag_asian_geom_greeks (s30sbc) computes the price of an Asian geometric continuous average-rate option, together with the Greeks or sensitivities, which are the partial derivatives of the option price with respect to certain of the other input parameters. The annual volatility, σ, risk-free rate, r, and cost of carry, b, are constants (see Kemna and Vorst (1990)). For a given strike price, X, the price of a call option with underlying price, S, and time to expiry, T, is

$$P_{\mathrm{call}} = Se^{(\bar{b}-r)T}\Phi(\bar{d}_1) - Xe^{-rT}\Phi(\bar{d}_2),$$

and the corresponding put option price is

$$P_{\mathrm{put}} = Xe^{-rT}\Phi(-\bar{d}_2) - Se^{(\bar{b}-r)T}\Phi(-\bar{d}_1),$$

where

$$\bar{d}_1 = \frac{\ln(S/X) + (\bar{b} + \bar{\sigma}^2/2)T}{\bar{\sigma}\sqrt{T}}$$

and

$$\bar{d}_2 = \bar{d}_1 - \bar{\sigma}\sqrt{T},$$

with

$$\bar{\sigma} = \frac{\sigma}{\sqrt{3}}, \quad \bar{b} = \frac{1}{2}\left(b - \frac{\sigma^2}{6}\right).$$

Φ is the cumulative Normal distribution function,

$$\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} \exp(-y^2/2)\,dy.$$

The option price $P_{ij} = P(X = X_i, T = T_j)$ is computed for each strike price in a set X_i, $i = 1, 2, \ldots, m$, and for each expiry time in a set T_j, $j = 1, 2, \ldots, n$.

4 References

Kemna A and Vorst A (1990) A pricing method for options based on average asset values *Journal of Banking and Finance* **14** 113–129

5 Arguments

1: **order** – Nag_OrderType *Input*

On entry: the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag_RowMajor. See Section 3.2.1.3 in the Essential Introduction for a more detailed explanation of the use of this argument.

Constraint: **order** = Nag_RowMajor or Nag_ColMajor.

2: **option** – Nag_CallPut *Input*

On entry: determines whether the option is a call or a put.

option = Nag_Call
A call; the holder has a right to buy.

option = Nag_Put
A put; the holder has a right to sell.

Constraint: **option** = Nag_Call or Nag_Put.

3: **m** – Integer *Input*

On entry: the number of strike prices to be used.

Constraint: **m** ≥ 1.

4: **n** – Integer *Input*

On entry: the number of times to expiry to be used.

Constraint: **n** ≥ 1.

5: **x**[**m**] – const double *Input*

On entry: **x**$[i-1]$ must contain X_i, the ith strike price, for $i = 1, 2, \ldots,$ **m**.

Constraint: **x**$[i-1] \geq z$ and **x**$[i-1] \leq 1/z$, where $z =$ nag_real_safe_small_number, the safe range parameter, for $i = 1, 2, \ldots,$ **m**.

6: **s** – double *Input*

On entry: S, the price of the underlying asset.

Constraint: **s** $\geq z$ and **s** $\leq 1.0/z$, where $z =$ nag_real_safe_small_number, the safe range parameter.

7: **t**[**n**] – const double *Input*

On entry: **t**$[i-1]$ must contain T_i, the ith time, in years, to expiry, for $i = 1, 2, \ldots,$ **n**.

Constraint: **t**$[i-1] \geq z$, where $z =$ nag_real_safe_small_number, the safe range parameter, for $i = 1, 2, \ldots,$ **n**.

8: **sigma** – double *Input*

On entry: σ, the volatility of the underlying asset. Note that a rate of 15% should be entered as 0.15.

Constraint: **sigma** > 0.0.

9: **r** – double *Input*

On entry: r, the annual risk-free interest rate, continuously compounded. Note that a rate of 5% should be entered as 0.05.

Constraint: $\mathbf{r} \geq 0.0$.

10: **b** – double *Input*

On entry: b, the annual cost of carry rate. Note that a rate of 8% should be entered as 0.08.

11: $\mathbf{p}[\mathbf{m} \times \mathbf{n}]$ – double *Output*

Note: where $\mathbf{P}(i, j)$ appears in this document, it refers to the array element

$\mathbf{p}[(j-1) \times \mathbf{m} + i - 1]$ when **order** = Nag_ColMajor;
$\mathbf{p}[(i-1) \times \mathbf{n} + j - 1]$ when **order** = Nag_RowMajor.

On exit: $\mathbf{P}(i, j)$ contains P_{ij}, the option price evaluated for the strike price $\mathbf{x}_i$ at expiry $\mathbf{t}_j$ for $i = 1, 2, \ldots, \mathbf{m}$ and $j = 1, 2, \ldots, \mathbf{n}$.

12: $\mathbf{delta}[\mathbf{m} \times \mathbf{n}]$ – double *Output*

Note: where $\mathbf{DELTA}(i, j)$ appears in this document, it refers to the array element

$\mathbf{delta}[(j-1) \times \mathbf{m} + i - 1]$ when **order** = Nag_ColMajor;
$\mathbf{delta}[(i-1) \times \mathbf{n} + j - 1]$ when **order** = Nag_RowMajor.

On exit: the $m \times n$ array **delta** contains the sensitivity, $\frac{\partial P}{\partial S}$, of the option price to change in the price of the underlying asset.

13: $\mathbf{gamma}[\mathbf{m} \times \mathbf{n}]$ – double *Output*

Note: the (i, j)th element of the matrix is stored in

$\mathbf{gamma}[(j-1) \times \mathbf{m} + i - 1]$ when **order** = Nag_ColMajor;
$\mathbf{gamma}[(i-1) \times \mathbf{n} + j - 1]$ when **order** = Nag_RowMajor.

On exit: the $m \times n$ array **gamma** contains the sensitivity, $\frac{\partial^2 P}{\partial S^2}$, of **delta** to change in the price of the underlying asset.

14: $\mathbf{vega}[\mathbf{m} \times \mathbf{n}]$ – double *Output*

Note: where $\mathbf{VEGA}(i, j)$ appears in this document, it refers to the array element

$\mathbf{vega}[(j-1) \times \mathbf{m} + i - 1]$ when **order** = Nag_ColMajor;
$\mathbf{vega}[(i-1) \times \mathbf{n} + j - 1]$ when **order** = Nag_RowMajor.

On exit: $\mathbf{VEGA}(i, j)$, contains the first-order Greek measuring the sensitivity of the option price P_{ij} to change in the volatility of the underlying asset, i.e., $\frac{\partial P_{ij}}{\partial \sigma}$, for $i = 1, 2, \ldots, \mathbf{m}$ and $j = 1, 2, \ldots, \mathbf{n}$.

15: $\mathbf{theta}[\mathbf{m} \times \mathbf{n}]$ – double *Output*

Note: where $\mathbf{THETA}(i, j)$ appears in this document, it refers to the array element

$\mathbf{theta}[(j-1) \times \mathbf{m} + i - 1]$ when **order** = Nag_ColMajor;
$\mathbf{theta}[(i-1) \times \mathbf{n} + j - 1]$ when **order** = Nag_RowMajor.

On exit: $\mathbf{THETA}(i, j)$, contains the first-order Greek measuring the sensitivity of the option price P_{ij} to change in time, i.e., $-\frac{\partial P_{ij}}{\partial T}$, for $i = 1, 2, \ldots, \mathbf{m}$ and $j = 1, 2, \ldots, \mathbf{n}$, where $b = r - q$.

16: **rho**[**m** × **n**] – double *Output*

Note: where **RHO**(i,j) appears in this document, it refers to the array element

rho$[(j-1)\times$ **m** $+ i - 1]$ when **order** = Nag_ColMajor;
rho$[(i-1)\times$ **n** $+ j - 1]$ when **order** = Nag_RowMajor.

On exit: **RHO**(i,j), contains the first-order Greek measuring the sensitivity of the option price P_{ij} to change in the annual risk-free interest rate, i.e., $-\frac{\partial P_{ij}}{\partial r}$, for $i = 1, 2, \ldots,$ **m** and $j = 1, 2, \ldots,$ **n**.

17: **crho**[**m** × **n**] – double *Output*

Note: the (i,j)th element of the matrix is stored in

crho$[(j-1)\times$ **m** $+ i - 1]$ when **order** = Nag_ColMajor;
crho$[(i-1)\times$ **n** $+ j - 1]$ when **order** = Nag_RowMajor.

On exit: **DELTA**(i,j), contains the first-order Greek measuring the sensitivity of the option price P_{ij} to change in the price of the underlying asset, i.e., $-\frac{\partial P_{ij}}{\partial S}$, for $i = 1, 2, \ldots,$ **m** and $j = 1, 2, \ldots,$ **n**.

18: **vanna**[**m** × **n**] – double *Output*

Note: where **VANNA**(i,j) appears in this document, it refers to the array element

vanna$[(j-1)\times$ **m** $+ i - 1]$ when **order** = Nag_ColMajor;
vanna$[(i-1)\times$ **n** $+ j - 1]$ when **order** = Nag_RowMajor.

On exit: **VANNA**(i,j), contains the second-order Greek measuring the sensitivity of the first-order Greek Δ_{ij} to change in the volatility of the asset price, i.e., $-\frac{\partial \Delta_{ij}}{\partial T} = -\frac{\partial^2 P_{ij}}{\partial S \partial \sigma}$, for $i = 1, 2, \ldots,$ **m** and $j = 1, 2, \ldots,$ **n**.

19: **charm**[**m** × **n**] – double *Output*

Note: where **CHARM**(i,j) appears in this document, it refers to the array element

charm$[(j-1)\times$ **m** $+ i - 1]$ when **order** = Nag_ColMajor;
charm$[(i-1)\times$ **n** $+ j - 1]$ when **order** = Nag_RowMajor.

On exit: **CHARM**(i,j), contains the second-order Greek measuring the sensitivity of the first-order Greek Δ_{ij} to change in the time, i.e., $-\frac{\partial \Delta_{ij}}{\partial T} = -\frac{\partial^2 P_{ij}}{\partial S \partial T}$, for $i = 1, 2, \ldots,$ **m** and $j = 1, 2, \ldots,$ **n**.

20: **speed**[**m** × **n**] – double *Output*

Note: where **SPEED**(i,j) appears in this document, it refers to the array element

speed$[(j-1)\times$ **m** $+ i - 1]$ when **order** = Nag_ColMajor;
speed$[(i-1)\times$ **n** $+ j - 1]$ when **order** = Nag_RowMajor.

On exit: **SPEED**(i,j), contains the third-order Greek measuring the sensitivity of the second-order Greek Γ_{ij} to change in the price of the underlying asset, i.e., $-\frac{\partial \Gamma_{ij}}{\partial S} = -\frac{\partial^3 P_{ij}}{\partial S^3}$, for $i = 1, 2, \ldots,$ **m** and $j = 1, 2, \ldots,$ **n**.

21: **colour**[**m** × **n**] – double *Output*

Note: where **COLOUR**(i,j) appears in this document, it refers to the array element

colour$[(j-1)\times$ **m** $+ i - 1]$ when **order** = Nag_ColMajor;
colour$[(i-1)\times$ **n** $+ j - 1]$ when **order** = Nag_RowMajor.

On exit: **COLOUR**(i,j), contains the third-order Greek measuring the sensitivity of the second-order Greek Γ_{ij} to change in the time, i.e., $-\frac{\partial \Gamma_{ij}}{\partial T} = -\frac{\partial^3 P_{ij}}{\partial S \partial T}$, for $i = 1, 2, \ldots,$ **m** and $j = 1, 2, \ldots,$ **n**.

22: **zomma**[**m** × **n**] – double *Output*

Note: where **ZOMMA**(i, j) appears in this document, it refers to the array element

zomma$[(j - 1) \times$ **m** $+ i - 1]$ when **order** = Nag_ColMajor;
zomma$[(i - 1) \times$ **n** $+ j - 1]$ when **order** = Nag_RowMajor.

On exit: **ZOMMA**(i, j), contains the third-order Greek measuring the sensitivity of the second-order Greek Γ_{ij} to change in the volatility of the underlying asset, i.e., $-\frac{\partial \Gamma_{ij}}{\partial \sigma} = -\frac{\partial^3 P_{ij}}{\partial S^2 \partial \sigma}$, for $i = 1, 2, \ldots,$ **m** and $j = 1, 2, \ldots,$ **n**.

23: **vomma**[**m** × **n**] – double *Output*

Note: where **VOMMA**(i, j) appears in this document, it refers to the array element

vomma$[(j - 1) \times$ **m** $+ i - 1]$ when **order** = Nag_ColMajor;
vomma$[(i - 1) \times$ **n** $+ j - 1]$ when **order** = Nag_RowMajor.

On exit: **VOMMA**(i, j), contains the second-order Greek measuring the sensitivity of the first-order Greek Δ_{ij} to change in the volatility of the underlying asset, i.e., $-\frac{\partial \Delta_{ij}}{\partial \sigma} = -\frac{\partial^2 P_{ij}}{\partial \sigma^2}$, for $i = 1, 2, \ldots,$ **m** and $j = 1, 2, \ldots,$ **n**.

24: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.

NE_BAD_PARAM

On entry, argument ⟨*value*⟩ had an illegal value.

NE_INT

On entry, **m** = ⟨*value*⟩.
Constraint: **m** ≥ 1.

On entry, **n** = ⟨*value*⟩.
Constraint: **n** ≥ 1.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

NE_REAL

On entry, **r** = ⟨*value*⟩.
Constraint: **r** ≥ 0.0.

On entry, **s** = ⟨*value*⟩.
Constraint: **s** ≥ ⟨*value*⟩ and **s** ≤ ⟨*value*⟩.

On entry, **sigma** = ⟨*value*⟩.
Constraint: **sigma** > 0.0.

NE_REAL_ARRAY

On entry, **t**[⟨*value*⟩] = ⟨*value*⟩.
Constraint: **t**[*i*] ≥ ⟨*value*⟩.

On entry, $\mathbf{x}[\langle value\rangle] = \langle value\rangle$.
Constraint: $\mathbf{x}[i] \geq \langle value\rangle$ and $\mathbf{x}[i] \leq \langle value\rangle$.

7 Accuracy

The accuracy of the output is dependent on the accuracy of the cumulative Normal distribution function, Φ. This is evaluated using a rational Chebyshev expansion, chosen so that the maximum relative error in the expansion is of the order of the ***machine precision*** (see nag_cumul_normal (s15abc) and nag_erfc (s15adc)). An accuracy close to ***machine precision*** can generally be expected.

8 Parallelism and Performance

nag_asian_geom_greeks (s30sbc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

Please consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

None.

10 Example

This example computes the price of an Asian geometric continuous average-rate call with a time to expiry of 3 months, a stock price of 80 and a strike price of 97. The risk-free interest rate is 5% per year, the cost of carry is 8% and the volatility is 20% per year.

10.1 Program Text

```
/* nag_asian_geom_greeks (s30sbc) Example Program.
 *
 * Copyright 2009, Numerical Algorithms Group.
 *
 * Mark 9, 2009.
 */
#include <stdio.h>
#include <math.h>
#include <string.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
  /* Integer scalar and array declarations */
  Integer       exit_status = 0;
  Integer       i, j, m, n;
  NagError      fail;
  Nag_CallPut   putnum;
  /* Double scalar and array declarations */
  double        b, r, s, sigma;
  double        *charm = 0, *colour = 0, *crho = 0, *delta = 0, *gamma = 0;
  double        *p = 0, *rho = 0, *speed = 0, *t = 0, *theta = 0, *vanna = 0;
  double        *vega = 0, *vomma = 0, *x = 0, *zomma = 0;
  /* Character scalar and array declarations */
  char          put[8+1];
  Nag_OrderType order;

  INIT_FAIL(fail);

  printf("nag_asian_geom_greeks (s30sbc) Example Program Results\n");
  printf("Asian Option: Geometric Continuous Average-Rate\n\n");
  /* Skip heading in data file */
```

```
scanf("%*[^\n] ");
/* Read put */
scanf("%8s%*[^\n] ", put);
/*
 * nag_enum_name_to_value (x04nac).
 * Converts NAG enum member name to value
 */
putnum = (Nag_CallPut) nag_enum_name_to_value(put);
/* Read s, sigma, r, b */
scanf("%lf%lf%lf%lf%*[^\n] ", &s, &sigma, &r, &b);
/* Read m, n */
scanf("%ld%ld%*[^\n] ", &m, &n);
  #ifdef NAG_COLUMN_MAJOR
  #define CHARM(I, J)  charm[(J-1)*m + I-1]
  #define COLOUR(I, J) colour[(J-1)*m + I-1]
  #define CRHO(I, J)   crho[(J-1)*m + I-1]
  #define DELTA(I, J)  delta[(J-1)*m + I-1]
  #define GAMMA(I, J)  gamma[(J-1)*m + I-1]
  #define P(I, J)      p[(J-1)*m + I-1]
  #define RHO(I, J)    rho[(J-1)*m + I-1]
  #define SPEED(I, J)  speed[(J-1)*m + I-1]
  #define THETA(I, J)  theta[(J-1)*m + I-1]
  #define VANNA(I, J)  vanna[(J-1)*m + I-1]
  #define VEGA(I, J)   vega[(J-1)*m + I-1]
  #define VOMMA(I, J)  vomma[(J-1)*m + I-1]
  #define ZOMMA(I, J)  zomma[(J-1)*m + I-1]
order = Nag_ColMajor;
  #else
  #define CHARM(I, J)  charm[(I-1)*n + J-1]
  #define COLOUR(I, J) colour[(I-1)*n + J-1]
  #define CRHO(I, J)   crho[(I-1)*n + J-1]
  #define DELTA(I, J)  delta[(I-1)*n + J-1]
  #define GAMMA(I, J)  gamma[(I-1)*n + J-1]
  #define P(I, J)      p[(I-1)*n + J-1]
  #define RHO(I, J)    rho[(I-1)*n + J-1]
  #define SPEED(I, J)  speed[(I-1)*n + J-1]
  #define THETA(I, J)  theta[(I-1)*n + J-1]
  #define VANNA(I, J)  vanna[(I-1)*n + J-1]
  #define VEGA(I, J)   vega[(I-1)*n + J-1]
  #define VOMMA(I, J)  vomma[(I-1)*n + J-1]
  #define ZOMMA(I, J)  zomma[(I-1)*n + J-1]
order = Nag_RowMajor;
  #endif
if (!(charm = NAG_ALLOC(m*n, double)) ||
    !(colour = NAG_ALLOC(m*n, double)) ||
    !(crho = NAG_ALLOC(m*n, double)) ||
    !(delta = NAG_ALLOC(m*n, double)) ||
    !(gamma = NAG_ALLOC(m*n, double)) ||
    !(p = NAG_ALLOC(m*n, double)) ||
    !(rho = NAG_ALLOC(m*n, double)) ||
    !(speed = NAG_ALLOC(m*n, double)) ||
    !(t = NAG_ALLOC(n, double)) ||
    !(theta = NAG_ALLOC(m*n, double)) ||
    !(vanna = NAG_ALLOC(m*n, double)) ||
    !(vega = NAG_ALLOC(m*n, double)) ||
    !(vomma = NAG_ALLOC(m*n, double)) ||
    !(x = NAG_ALLOC(m, double)) ||
    !(zomma = NAG_ALLOC(m*n, double)))
  {
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
  }
/* Read array of strike/exercise prices, X */
for (i = 0; i < m; i++)
  scanf("%lf ", &x[i]);
scanf("%*[^\n] ");
/* Read array of times to expiry */
for (i = 0; i < n; i++)
  scanf("%lf ", &t[i]);
scanf("%*[^\n] ");
```

```
  /*
   * nag_asian_geom_greeks (s30sbc)
   * Asian option: geometric continuous average rate pricing formula
   * with Greeks
   */
  nag_asian_geom_greeks(order, putnum, m, n, x, s, t, sigma, r, b, p,
                        delta, gamma, vega, theta, rho, crho, vanna,
                        charm, speed, colour, zomma, vomma, &fail);
  if (fail.code != NE_NOERROR)
    {
      printf("Error from nag_asian_geom_greeks (s30sbc).\n%s\n",
              fail.message);
      exit_status = 1;
      goto END;
    }
  if (putnum == Nag_Call)
    printf("%s\n\n", "Asian Call :");
  else if (putnum == Nag_Put)
    printf("%s\n\n", "Asian Put :");
  printf("  Spot          = %8.4f\n", s);
  printf("  Volatility    = %8.4f\n", sigma);
  printf("  Rate          = %8.4f\n", r);
  printf("  Cost of carry = %8.4f\n", b);
  printf("\n");
  for (j = 1; j <= n; j++)
    {
      printf("\n  Time to Expiry :    %8.4f\n", t[j-1]);
      printf("  Strike    Price    Delta    Gamma     Vega    "
              "Theta      Rho     CRho\n");
      for (i = 1; i <= m; i++)
        printf("%8.4f %8.4f %8.4f %8.4f %8.4f %8.4f %8.4f %8.4f\n",
                x[i-1], P(i, j), DELTA(i, j), GAMMA(i, j), VEGA(i, j),
                THETA(i, j), RHO(i, j), CRHO(i, j));
      printf("                      Vanna    Charm    Speed   "
              "Colour    Zomma    Vomma\n");
      for (i = 1; i <= m; i++)
        printf("%26.4f %8.4f %8.4f %8.4f %8.4f %8.4f\n", VANNA(i, j),
                CHARM(i, j), SPEED(i, j), COLOUR(i, j), ZOMMA(i, j),
                VOMMA(i, j));
    }

 END:
  NAG_FREE(charm);
  NAG_FREE(colour);
  NAG_FREE(crho);
  NAG_FREE(delta);
  NAG_FREE(gamma);
  NAG_FREE(p);
  NAG_FREE(rho);
  NAG_FREE(speed);
  NAG_FREE(t);
  NAG_FREE(theta);
  NAG_FREE(vanna);
  NAG_FREE(vega);
  NAG_FREE(vomma);
  NAG_FREE(x);
  NAG_FREE(zomma);

  return exit_status;
}
```

10.2 Program Data

```
nag_asian_geom_greeks (s30sbc) Example Program Data
 Nag_Call            : Nag_Call or Nag_Put
 80.0 0.2 0.05 0.08 : s, sigma, r, b
 1 1                 : m, n
 97.0                : X(I), I = 1,2,...m
 0.25                : T(I), I = 1,2,...n
```

10.3 Program Results

```
nag_asian_geom_greeks (s30sbc) Example Program Results
Asian Option: Geometric Continuous Average-Rate

Asian Call :

  Spot          =  80.0000
  Volatility    =   0.2000
  Rate          =   0.0500
  Cost of carry =   0.0800


  Time to Expiry :       0.2500
  Strike     Price     Delta     Gamma      Vega     Theta       Rho      CRho
 97.0000    0.0010    0.0008    0.0006    0.0638   -0.0281    0.0079    0.0081
                       Vanna     Charm     Speed    Colour     Zomma     Vomma
                      0.0443   -0.0196    0.0004   -0.0122    0.0272    3.1893
```