

NAG Library Function Document

nag_kelvin_kei (s19adc)

1 Purpose

nag_kelvin_kei (s19adc) returns a value for the Kelvin function $\text{kei } x$.

2 Specification

```
#include <nag.h>
#include <nags.h>
double nag_kelvin_kei (double x, NagError *fail)
```

3 Description

nag_kelvin_kei (s19adc) evaluates an approximation to the Kelvin function $\text{kei } x$.

Note: for $x < 0$ the function is undefined, so we need only consider $x \geq 0$.

The function is based on several Chebyshev expansions:

For $0 \leq x \leq 1$,

$$\text{kei } x = -\frac{\pi}{4}f(t) + \frac{x^2}{4}[-g(t)\log(x) + v(t)]$$

where $f(t)$, $g(t)$ and $v(t)$ are expansions in the variable $t = 2x^4 - 1$;

For $1 < x \leq 3$,

$$\text{kei } x = \exp\left(-\frac{9}{8}x\right)u(t)$$

where $u(t)$ is an expansion in the variable $t = x - 2$;

For $x > 3$,

$$\text{kei } x = \sqrt{\frac{\pi}{2x}}e^{-x/\sqrt{2}}\left[\left(1 + \frac{1}{x}\right)c(t)\sin\beta + \frac{1}{x}d(t)\cos\beta\right]$$

where $\beta = \frac{x}{\sqrt{2}} + \frac{\pi}{8}$, and $c(t)$ and $d(t)$ are expansions in the variable $t = \frac{6}{x} - 1$.

For $x < 0$, the function is undefined, and hence the function fails and returns zero.

When x is sufficiently close to zero, the result is computed as

$$\text{kei } x = -\frac{\pi}{4} + \left(1 - \gamma - \log\left(\frac{x}{2}\right)\right)\frac{x^2}{4}$$

and when x is even closer to zero simply as

$$\text{kei } x = -\frac{\pi}{4}.$$

For large x , $\text{kei } x$ is asymptotically given by $\sqrt{\frac{\pi}{2x}}e^{-x/\sqrt{2}}$ and this becomes so small that it cannot be computed without underflow and the function fails.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Arguments

- 1: **x** – double *Input*
On entry: the argument x of the function.
Constraint: $x \geq 0.0$.
- 2: **fail** – NagError * *Input/Output*
 The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

NE_REAL_ARG_GT

On entry, $x = \langle \text{value} \rangle$. The function returns zero.
 Constraint: $x \leq \langle \text{value} \rangle$.
 x is too large, the result underflows and the function returns zero.

NE_REAL_ARG_LT

On entry, $x = \langle \text{value} \rangle$.
 Constraint: $x \geq 0.0$.
 The function is undefined and returns zero.

7 Accuracy

Let E be the absolute error in the result, and δ be the relative error in the argument. If δ is somewhat larger than the machine representation error, then we have:

$$E \simeq \left| \frac{x}{\sqrt{2}} (-\text{ker}_1 x + \text{kei}_1 x) \right| \delta.$$

For small x , errors are attenuated by the function and hence are limited by the *machine precision*.

For medium and large x , the error behaviour, like the function itself, is oscillatory and hence only absolute accuracy of the function can be maintained. For this range of x , the amplitude of the absolute error decays like $\sqrt{\frac{\pi x}{2}} e^{-x/\sqrt{2}}$, which implies a strong attenuation of error. Eventually, $\text{kei } x$, which is asymptotically given by $\sqrt{\frac{\pi}{2x}} e^{-x/\sqrt{2}}$, becomes so small that it cannot be calculated without causing underflow and therefore the function returns zero. Note that for large x , the errors are dominated by those of the standard math library function \exp .

8 Parallelism and Performance

Not applicable.

9 Further Comments

Underflow may occur for a few values of x close to the zeros of $\text{kei } x$, below the limit which causes a failure with `fail.code = NE_REAL_ARG_GT`.

10 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

10.1 Program Text

```

/* nag_kelvin_kei (s19adc) Example Program.
 *
 * Copyright 1990 Numerical Algorithms Group.
 *
 * Mark 2 revised, 1992.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
    Integer  exit_status = 0;
    double   x, y;
    NagError fail;

    INIT_FAIL(fail);

    /* Skip heading in data file */
    scanf("%*[\n]");
    printf("nag_kelvin_kei (s19adc) Example Program Results\n");
    printf("      x          y\n");
    while (scanf("%lf", &x) != EOF)
    {
        /* nag_kelvin_kei (s19adc).
         * Kelvin function kei x
         */
        y = nag_kelvin_kei(x, &fail);
        if (fail.code != NE_NOERROR)
        {
            printf("Error from nag_kelvin_kei (s19adc).\n%s\n",
                fail.message);
            exit_status = 1;
            goto END;
        }
        printf("%12.3e%12.3e\n", x, y);
    }

    END:
    return exit_status;
}

```

10.2 Program Data

```

nag_kelvin_kei (s19adc) Example Program Data
      0.0
      0.1
      1.0
      2.5
      5.0
     10.0
     15.0

```

10.3 Program Results

nag_kelvin_kei (s19adc) Example Program Results

x	y
0.000e+00	-7.854e-01
1.000e-01	-7.769e-01
1.000e+00	-4.950e-01
2.500e+00	-1.107e-01
5.000e+00	1.119e-02
1.000e+01	-3.075e-04
1.500e+01	7.963e-06
