

NAG Library Function Document

nag_zpptrs (f07gsc)

1 Purpose

nag_zpptrs (f07gsc) solves a complex Hermitian positive definite system of linear equations with multiple right-hand sides,

$$AX = B,$$

where A has been factorized by nag_zpptrf (f07grc), using packed storage.

2 Specification

```
#include <nag.h>
#include <nagf07.h>

void nag_zpptrs (Nag_OrderType order, Nag_UploType uplo, Integer n,
                Integer nrhs, const Complex ap[], Complex b[], Integer pdb,
                NagError *fail)
```

3 Description

nag_zpptrs (f07gsc) is used to solve a complex Hermitian positive definite system of linear equations $AX = B$, the function must be preceded by a call to nag_zpptrf (f07grc) which computes the Cholesky factorization of A , using packed storage. The solution X is computed by forward and backward substitution.

If **uplo** = Nag_Upper, $A = U^H U$, where U is upper triangular; the solution X is computed by solving $U^H Y = B$ and then $UX = Y$.

If **uplo** = Nag_Lower, $A = LL^H$, where L is lower triangular; the solution X is computed by solving $LY = B$ and then $L^H X = Y$.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Arguments

1: **order** – Nag_OrderType *Input*

On entry: the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag_RowMajor. See Section 3.2.1.3 in the Essential Introduction for a more detailed explanation of the use of this argument.

Constraint: **order** = Nag_RowMajor or Nag_ColMajor.

2: **uplo** – Nag_UploType *Input*

On entry: specifies how A has been factorized.

uplo = Nag_Upper
 $A = U^H U$, where U is upper triangular.

uplo = Nag_Lower
 $A = LL^H$, where L is lower triangular.
 Constraint: **uplo** = Nag_Upper or Nag_Lower.

- 3: **n** – Integer *Input*
 On entry: n , the order of the matrix A .
 Constraint: $n \geq 0$.
- 4: **nrhs** – Integer *Input*
 On entry: r , the number of right-hand sides.
 Constraint: **nrhs** ≥ 0 .
- 5: **ap**[*dim*] – const Complex *Input*
Note: the dimension, *dim*, of the array **ap** must be at least $\max(1, n \times (n + 1)/2)$.
 On entry: the Cholesky factor of A stored in packed form, as returned by nag_zpptrf (f07grc).
- 6: **b**[*dim*] – Complex *Input/Output*
Note: the dimension, *dim*, of the array **b** must be at least
 $\max(1, \mathbf{pdb} \times \mathbf{nrhs})$ when **order** = Nag_ColMajor;
 $\max(1, n \times \mathbf{pdb})$ when **order** = Nag_RowMajor.
 The (i, j)th element of the matrix B is stored in
 $\mathbf{b}[(j - 1) \times \mathbf{pdb} + i - 1]$ when **order** = Nag_ColMajor;
 $\mathbf{b}[(i - 1) \times \mathbf{pdb} + j - 1]$ when **order** = Nag_RowMajor.
 On entry: the n by r right-hand side matrix B .
 On exit: the n by r solution matrix X .
- 7: **pdb** – Integer *Input*
 On entry: the stride separating row or column elements (depending on the value of **order**) in the array **b**.
 Constraints:
 if **order** = Nag_ColMajor, **pdb** $\geq \max(1, n)$;
 if **order** = Nag_RowMajor, **pdb** $\geq \max(1, \mathbf{nrhs})$.
- 8: **fail** – NagError * *Input/Output*
 The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.

NE_BAD_PARAM

On entry, argument $\langle value \rangle$ had an illegal value.

NE_INT

On entry, $n = \langle value \rangle$.
 Constraint: $n \geq 0$.

On entry, **nrhs** = $\langle value \rangle$.

Constraint: **nrhs** ≥ 0 .

On entry, **pdb** = $\langle value \rangle$.

Constraint: **pdb** > 0 .

NE_INT_2

On entry, **pdb** = $\langle value \rangle$ and **n** = $\langle value \rangle$.

Constraint: **pdb** $\geq \max(1, \mathbf{n})$.

On entry, **pdb** = $\langle value \rangle$ and **nrhs** = $\langle value \rangle$.

Constraint: **pdb** $\geq \max(1, \mathbf{nrhs})$.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

7 Accuracy

For each right-hand side vector b , the computed solution x is the exact solution of a perturbed system of equations $(A + E)x = b$, where

if **uplo** = Nag_Upper, $|E| \leq c(n)\epsilon|U^H||U|$;

if **uplo** = Nag_Lower, $|E| \leq c(n)\epsilon|L||L^H|$,

$c(n)$ is a modest linear function of n , and ϵ is the *machine precision*.

If \hat{x} is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x - \hat{x}\|_{\infty}}{\|x\|_{\infty}} \leq c(n) \text{cond}(A, x)\epsilon$$

where $\text{cond}(A, x) = \frac{\|A^{-1}\|_{\infty}\|A\|_{\infty}\|x\|_{\infty}}{\|x\|_{\infty}} \leq \text{cond}(A) = \frac{\|A^{-1}\|_{\infty}\|A\|_{\infty}}{\|x\|_{\infty}} \leq \kappa_{\infty}(A)$.

Note that $\text{cond}(A, x)$ can be much smaller than $\text{cond}(A)$.

Forward and backward error bounds can be computed by calling `nag_zpprfs` (f07gvc), and an estimate for $\kappa_{\infty}(A)$ ($= \kappa_1(A)$) can be obtained by calling `nag_zppcon` (f07guc).

8 Parallelism and Performance

`nag_zpptrs` (f07gsc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

`nag_zpptrs` (f07gsc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

The total number of real floating-point operations is approximately $8n^2r$.

This function may be followed by a call to `nag_zpprfs` (f07gvc) to refine the solution and return an error estimate.

The real analogue of this function is `nag_dppttrs` (f07gec).

10 Example

This example solves the system of equations $AX = B$, where

$$A = \begin{pmatrix} 3.23 + 0.00i & 1.51 - 1.92i & 1.90 + 0.84i & 0.42 + 2.50i \\ 1.51 + 1.92i & 3.58 + 0.00i & -0.23 + 1.11i & -1.18 + 1.37i \\ 1.90 - 0.84i & -0.23 - 1.11i & 4.09 + 0.00i & 2.33 - 0.14i \\ 0.42 - 2.50i & -1.18 - 1.37i & 2.33 + 0.14i & 4.29 + 0.00i \end{pmatrix}$$

and

$$B = \begin{pmatrix} 3.93 - 6.14i & 1.48 + 6.58i \\ 6.17 + 9.42i & 4.65 - 4.75i \\ -7.17 - 21.83i & -4.91 + 2.29i \\ 1.99 - 14.38i & 7.64 - 10.79i \end{pmatrix}.$$

Here A is Hermitian positive definite, stored in packed form, and must first be factorized by `nag_zpptrf` (f07grc).

10.1 Program Text

```

/* nag_zpptrs (f07gsc) Example Program.
 *
 * Copyright 2001 Numerical Algorithms Group.
 *
 * Mark 7, 2001.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf07.h>
#include <nagx04.h>

int main(void)
{
    /* Scalars */
    Integer      ap_len, i, j, n, nrhs, pdb;
    Integer      exit_status = 0;
    NagError     fail;
    Nag_UploType uplo;
    Nag_OrderType order;
    /* Arrays */
    char         nag_enum_arg[40];
    Complex      *ap = 0, *b = 0;

#ifdef NAG_COLUMN_MAJOR
#define A_UPPER(I, J) ap[J*(J-1)/2 + I - 1]
#define A_LOWER(I, J) ap[(2*n-J)*(J-1)/2 + I - 1]
#define B(I, J)      b[(J-1)*pdb + I - 1]
    order = Nag_ColMajor;
#else
#define A_LOWER(I, J) ap[I*(I-1)/2 + J - 1]
#define A_UPPER(I, J) ap[(2*n-I)*(I-1)/2 + J - 1]
#define B(I, J)      b[(I-1)*pdb + J - 1]
    order = Nag_RowMajor;
#endif

    INIT_FAIL(fail);

    printf("nag_zpptrs (f07gsc) Example Program Results\n\n");

    /* Skip heading in data file */
    scanf("%*[\n] ");
    scanf("%ld%ld%*[\n] ", &n, &nrhs);
    ap_len = n * (n + 1)/2;
#ifdef NAG_COLUMN_MAJOR
    pdb = n;
#else

```

```

    pdb = nrhs;
#endif

/* Allocate memory */
if (!(ap = NAG_ALLOC(ap_len, Complex)) ||
    !(b = NAG_ALLOC(n * nrhs, Complex)))
{
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
}

/* Read A and B from data file */
scanf(" %39s%*[\n] ", nag_enum_arg);
/* nag_enum_name_to_value (x04nac).
 * Converts NAG enum member name to value
 */
uplo = (Nag_UploType) nag_enum_name_to_value(nag_enum_arg);

if (uplo == Nag_Upper)
{
    for (i = 1; i <= n; ++i)
    {
        for (j = i; j <= n; ++j)
            scanf(" ( %lf , %lf )", &A_UPPER(i, j).re,
                &A_UPPER(i, j).im);
    }
    scanf("%*[\n] ");
}
else
{
    for (i = 1; i <= n; ++i)
    {
        for (j = 1; j <= i; ++j)
            scanf(" ( %lf , %lf )", &A_LOWER(i, j).re,
                &A_LOWER(i, j).im);
    }
    scanf("%*[\n] ");
}
for (i = 1; i <= n; ++i)
{
    for (j = 1; j <= nrhs; ++j)
        scanf(" ( %lf , %lf )", &B(i, j).re, &B(i, j).im);
}
scanf("%*[\n] ");

/* Factorize A */
/* nag_zpptrf (f07grc).
 * Cholesky factorization of complex Hermitian
 * positive-definite matrix, packed storage
 */
nag_zpptrf(order, uplo, n, ap, &fail);
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_zpptrf (f07grc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
/* Compute solution */
/* nag_zpptrs (f07gsc).
 * Solution of complex Hermitian positive-definite system of
 * linear equations, multiple right-hand sides, matrix
 * already factorized by nag_zpptrf (f07grc), packed storage
 */
nag_zpptrs(order, uplo, n, nrhs, ap, b, pdb, &fail);
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_zpptrs (f07gsc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

```

```

/* Print solution */
/* nag_gen_complx_mat_print_comp (x04dbc).
 * Print complex general matrix (comprehensive)
 */
fflush(stdout);
nag_gen_complx_mat_print_comp(order, Nag_GeneralMatrix, Nag_NonUnitDiag, n,
                             nrhs, b, pdb, Nag_BracketForm, "%7.4f",
                             "Solution(s)", Nag_IntegerLabels,
                             0, Nag_IntegerLabels, 0, 80, 0, 0, &fail);
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_gen_complx_mat_print_comp (x04dbc).\n%s\n",
          fail.message);
    exit_status = 1;
    goto END;
}
END:
NAG_FREE(ap);
NAG_FREE(b);
return exit_status;
}

```

10.2 Program Data

```

nag_zpptrs (f07gsc) Example Program Data
  4  2                                     :Values of n and nrhs
  Nag_Lower                               :Value of uplo
(3.23, 0.00)
(1.51, 1.92) ( 3.58, 0.00)
(1.90,-0.84) (-0.23,-1.11) ( 4.09, 0.00)
(0.42,-2.50) (-1.18,-1.37) ( 2.33, 0.14) ( 4.29, 0.00) :End of matrix A
( 3.93, -6.14) ( 1.48,  6.58)
( 6.17,  9.42) ( 4.65, -4.75)
(-7.17,-21.83) (-4.91,  2.29)
( 1.99,-14.38) ( 7.64,-10.79)           :End of matrix B

```

10.3 Program Results

```

nag_zpptrs (f07gsc) Example Program Results

Solution(s)
           1           2
1 ( 1.0000,-1.0000) (-1.0000, 2.0000)
2 (-0.0000, 3.0000) ( 3.0000,-4.0000)
3 (-4.0000,-5.0000) (-2.0000, 3.0000)
4 ( 2.0000, 1.0000) ( 4.0000,-5.0000)

```
