NAG Toolbox

nag_specfun_kelvin_kei (s19ad)

1 Purpose

nag_specfun_kelvin_kei (s19ad) returns a value for the Kelvin function $\mathrm{kei}\, x$ via the function name.

2 Syntax

```
[result, ifail] = nag_specfun_kelvin_kei(x)
[result, ifail] = s19ad(x)
```

3 Description

nag_specfun_kelvin_kei (s19ad) evaluates an approximation to the Kelvin function $\mathrm{kei}\, x$.

Note: for $x < 0$ the function is undefined, so we need only consider $x \geq 0$.

The function is based on several Chebyshev expansions:

For $0 \leq x \leq 1$,

$$\mathrm{kei}\, x = -\frac{\pi}{4} f(t) + \frac{x^2}{4}[-g(t)\log(x) + v(t)]$$

where $f(t)$, $g(t)$ and $v(t)$ are expansions in the variable $t = 2x^4 - 1$;

For $1 < x \leq 3$,

$$\mathrm{kei}\, x = \exp\left(-\frac{9}{8}x\right) u(t)$$

where $u(t)$ is an expansion in the variable $t = x - 2$;

For $x > 3$,

$$\mathrm{kei}\, x = \sqrt{\frac{\pi}{2x}} e^{-x/\sqrt{2}} \left[\left(1 + \frac{1}{x}\right) c(t) \sin\beta + \frac{1}{x} d(t) \cos\beta\right]$$

where $\beta = \dfrac{x}{\sqrt{2}} + \dfrac{\pi}{8}$, and $c(t)$ and $d(t)$ are expansions in the variable $t = \dfrac{6}{x} - 1$.

For $x < 0$, the function is undefined, and hence the function fails and returns zero.

When x is sufficiently close to zero, the result is computed as

$$\mathrm{kei}\, x = -\frac{\pi}{4} + \left(1 - \gamma - \log\left(\frac{x}{2}\right)\right)\frac{x^2}{4}$$

and when x is even closer to zero simply as

$$\mathrm{kei}\, x = -\frac{\pi}{4}.$$

For large x, $\mathrm{kei}\, x$ is asymptotically given by $\sqrt{\dfrac{\pi}{2x}} e^{-x/\sqrt{2}}$ and this becomes so small that it cannot be computed without underflow and the function fails.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

5.1 Compulsory Input Parameters

1: **x** – REAL (KIND=nag_wp)

The argument x of the function.

Constraint: $\mathbf{x} \geq 0.0$.

5.2 Optional Input Parameters

None.

5.3 Output Parameters

1: **result**

The result of the function.

2: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, **x** is too large: the result underflows. On softfailure, the function returns zero.

ifail $= 2$

On entry, $\mathbf{x} < 0.0$: the function is undefined. On softfailure the function returns zero.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

Let E be the absolute error in the result, and δ be the relative error in the argument. If δ is somewhat larger than the machine representation error, then we have:

$$E \simeq \left| \frac{x}{\sqrt{2}} (-\mathrm{ker}_1\, x + \mathrm{kei}_1\, x) \right| \delta.$$

For small x, errors are attenuated by the function and hence are limited by the ***machine precision***.

For medium and large x, the error behaviour, like the function itself, is oscillatory and hence only absolute accuracy of the function can be maintained. For this range of x, the amplitude of the absolute error decays like $\sqrt{\frac{\pi x}{2}}e^{-x/\sqrt{2}}$, which implies a strong attenuation of error. Eventually, $\mathrm{kei}\,x$, which is asymptotically given by $\sqrt{\frac{\pi}{2x}}e^{-x/\sqrt{2}}$,becomes so small that it cannot be calculated without causing underflow and therefore the function returns zero. Note that for large x, the errors are dominated by those of the standard function exp.

8 Further Comments

Underflow may occur for a few values of x close to the zeros of $\mathrm{kei}\,x$, below the limit which causes a failure with **ifail** $= 1$.

9 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

```
function s19ad_example

fprintf('s19ad example results\n\n');

x = [0.1   1    2.5   5   10   15];
n = size(x,2);
result = x;

for j=1:n
  [result(j), ifail] = s19ad(x(j));
end

disp('      x          kei(x)');
fprintf('%12.3e%12.3e\n',[x; result]);
```

9.2 Program Results

```
s19ad example results

      x          kei(x)
   1.000e-01  -7.769e-01
   1.000e+00  -4.950e-01
   2.500e+00  -1.107e-01
   5.000e+00   1.119e-02
   1.000e+01  -3.075e-04
   1.500e+01   7.963e-06
```