

NAG Toolbox

nag_specfun_dawson (s15af)

1 Purpose

nag_specfun_dawson (s15af) returns a value for Dawson's Integral, $F(x)$, via the function name.

2 Syntax

```
[result, ifail] = nag_specfun_dawson(x)
[result, ifail] = s15af(x)
```

3 Description

nag_specfun_dawson (s15af) evaluates an approximation for Dawson's Integral

$$F(x) = e^{-x^2} \int_0^x e^{t^2} dt.$$

The function is based on two Chebyshev expansions:

For $0 < |x| \leq 4$,

$$F(x) = x \sum_{r=0}^l a_r T_r(t), \quad \text{where } t = 2\left(\frac{x}{4}\right)^2 - 1.$$

For $|x| > 4$,

$$F(x) = \frac{1}{x} \sum_{r=0}^l b_r T_r(t), \quad \text{where } t = 2\left(\frac{4}{x}\right)^2 - 1.$$

For $|x|$ near zero, $F(x) \simeq x$, and for $|x|$ large, $F(x) \simeq \frac{1}{2x}$. These approximations are used for those values of x for which the result is correct to *machine precision*.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

5.1 Compulsory Input Parameters

1: **x** – REAL (KIND=nag_wp)
The argument x of the function.

5.2 Optional Input Parameters

None.

5.3 Output Parameters

1: **result**
The result of the function.

2: **ifail** – INTEGER

ifail = 0 unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

There are no failure exits from this routine.

7 Accuracy

Let δ and ϵ be the relative errors in the argument and result respectively.

If δ is considerably greater than the *machine precision* (i.e., if δ is due to data errors etc.), then ϵ and δ are approximately related by:

$$\epsilon \simeq \left| \frac{x(1 - 2xF(x))}{F(x)} \right| \delta.$$

The following graph shows the behaviour of the error amplification factor $\left| \frac{x(1 - 2xF(x))}{F(x)} \right|$:

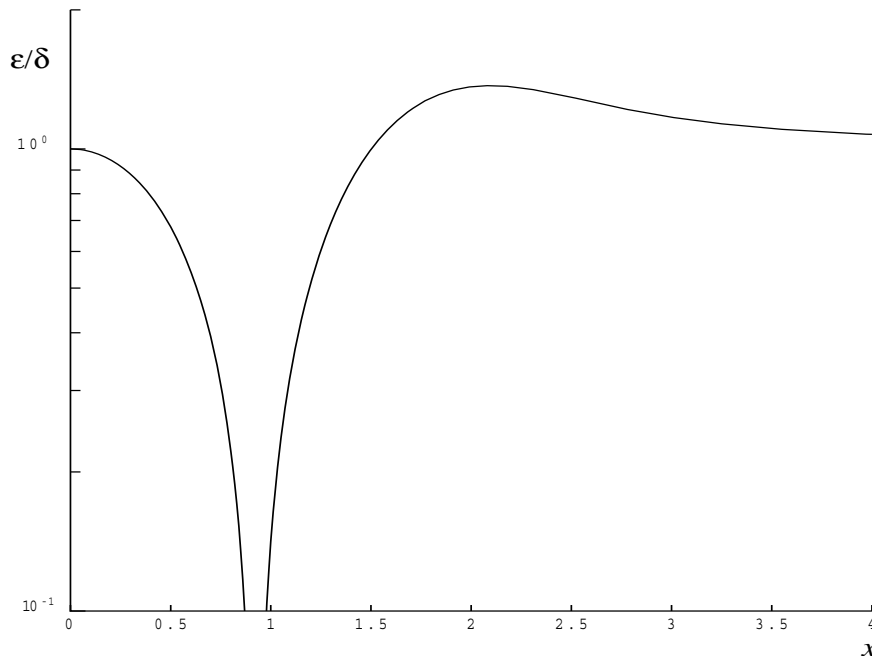


Figure 1

However if δ is of the same order as *machine precision*, then rounding errors could make ϵ somewhat larger than the above relation indicates. In fact ϵ will be largely independent of x or δ , but will be of the order of a few times the *machine precision*.

8 Further Comments

None.

9 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

```
function s15af_example
fprintf('s15af example results\n\n');
x = [-2   -0.5   1   1.5   2   5  10];
n = size(x,2);
result = x;
for j=1:n
    [result(j), ifail] = s15af(x(j));
end
disp('      x          F(x)');
fprintf('%12.3e%12.3e\n',[x; result]);
```

9.2 Program Results

```
s15af example results
      x          F(x)
-2.000e+00  -3.013e-01
-5.000e-01  -4.244e-01
 1.000e+00   5.381e-01
 1.500e+00   4.282e-01
 2.000e+00   3.013e-01
 5.000e+00   1.021e-01
 1.000e+01   5.025e-02
```
