NAG Toolbox

nag_specfun_beta_incomplete (s14cc)

1 Purpose
nag_specfun_beta_incomplete (s14cc) computes values for the incomplete beta function $I_x(a, b)$ and its complement $1 - I_x(a, b)$.

2 Syntax

```matlab
[w, w1, ifail] = nag_specfun_beta_incomplete(a, b, x)
[w, w1, ifail] = s14cc(a, b, x)
```

3 Description
nag_specfun_beta_incomplete (s14cc) evaluates the incomplete beta function and its complement in the normalized form

$$I_x(a, b) = \frac{1}{B(a, b)} \int_0^x t^{a-1} (1 - t)^{b-1} dt$$

$$1 - I_x(a, b) = I_y(b, a), \text{ where } y = 1 - x,$$

with

- $0 \leq x \leq 1$,
- $a \geq 0$ and $b \geq 0$,

and the beta function $B(a, b)$ is defined as $B(a, b) = \int_0^1 t^{a-1} (1 - t)^{b-1} dt = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$ where $\Gamma(y)$ is the gamma function.

Several methods are used to evaluate the functions depending on the arguments $a, b$ and $x$. The methods include Wise’s asymptotic expansion (see Wise (1950)) when $a > b$, continued fraction derived by DiDonato and Morris (1992) when $a, b > 1$, and power series when $b \leq 1$ or $b \times x \leq 0.7$. When both $a$ and $b$ are large, specifically $a, b \geq 15$, the DiDonato and Morris (1992) asymptotic expansion is employed for greater efficiency.

Once either $I_x(a, b)$ or $I_y(b, a)$ is computed, the other is obtained by subtraction from 1. In order to avoid loss of relative precision in this subtraction, the smaller of $I_x(a, b)$ and $I_y(b, a)$ is computed first. nag_specfun_beta_incomplete (s14cc) is derived from BRATIO in DiDonato and Morris (1992).

4 References

Wise M E (1950) The incomplete beta function as a contour integral and a quickly converging series for its inverse Biometrika 37 208–218
5 Parameters

5.1 Compulsory Input Parameters

1: $a$ – REAL (KIND=nag_wp)
   The argument $a$ of the function.
   Constraint: $a \geq 0.0$.

2: $b$ – REAL (KIND=nag_wp)
   The argument $b$ of the function.
   Constraints:
   - $b \geq 0.0$;
   - either $b \neq 0.0$ or $a \neq 0.0$.

3: $x$ – REAL (KIND=nag_wp)
   $x$, upper limit of integration.
   Constraints:
   - $0.0 \leq x \leq 1.0$;
   - either $x \neq 0.0$ or $a \neq 0.0$;
   - either $1 - x \neq 0.0$ or $b \neq 0.0$.

5.2 Optional Input Parameters

None.

5.3 Output Parameters

1: $w$ – REAL (KIND=nag_wp)
   The value of the incomplete beta function $I_x(a, b)$ evaluated from zero to $x$.

2: $w1$ – REAL (KIND=nag_wp)
   The value of the complement of the incomplete beta function $1 - I_x(a, b)$, i.e., the incomplete beta function evaluated from $x$ to one.

3: $\text{ifail}$ – INTEGER
   $\text{ifail} = 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

$\text{ifail} = 1$
   Constraint: $a \geq 0.0$.
   Constraint: $b \geq 0.0$.

$\text{ifail} = 2$
   On entry, $a$ and $b$ were zero.
   Constraint: $a$ or $b$ must be nonzero.

$\text{ifail} = 3$
   Constraint: $0.0 \leq x \leq 1.0$.
ifail = 4
    On entry, x and a were zero.
    Constraint: x or a must be nonzero.

ifail = 5
    On entry, 1.0 - x and b were zero.
    Constraint: 1.0 - x or b must be nonzero.

ifail = -99
    An unexpected error has been triggered by this routine. Please contact NAG.

ifail = -399
    Your licence key may have expired or may not have been installed correctly.

ifail = -999
    Dynamic memory allocation failed.

7 Accuracy

nag_specfun_beta_incomplete (s14cc) is designed to maintain relative accuracy for all arguments. For very tiny results (of the order of machine precision or less) some relative accuracy may be lost – loss of three or four decimal places has been observed in experiments. For other arguments full relative accuracy may be expected.

8 Further Comments

None.

9 Example

This example reads values of the arguments a and b from a file, evaluates the function and its complement for 10 different values of x and prints the results.

9.1 Program Text

    function s14cc_example

    fprintf('s14cc example results

    a = 5.3;
b = 10.1;
    fprintf('
    a b x Ix(a,b) 1-Ix(a,b)
    for x = 0.01:0.01:0.1
        [w, w1, ifail] = s14cc(a, b, x);
        fprintf('%6.2f%6.2f%6.2f%17.4e%17.4e
        end

9.2 Program Results

    s14cc example results

    a b x Ix(a,b) 1-Ix(a,b)
    5.30 10.10 0.01  6.4755e-08 1.0000e+00
    5.30 10.10 0.02  2.3613e-06 1.0000e+00
    5.30 10.10 0.03  1.8734e-05 9.9976e-01
    5.30 10.10 0.04  7.9575e-05 9.9992e-01
    5.30 10.10 0.05  2.3997e-04 9.9976e-01

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S – Approximations of Special Functions

s14cc
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