NAG Toolbox

nag_nonpar_concordance_kendall (g08da)

1 Purpose

nag_nonpar_concordance_kendall (g08da) calculates Kendall's coefficient of concordance on k independent rankings of n objects or individuals.

2 Syntax

```
[w, p, ifail] = nag_nonpar_concordance_kendall(x, k, 'n', n)
[w, p, ifail] = g08da(x, k, 'n', n)
```

3 Description

Kendall's coefficient of concordance measures the degree of agreement between k comparisons of n objects, the scores in the ith comparison being denoted by

$$x_{i1}, x_{i2}, \ldots, x_{in}.$$

The hypothesis under test, H_0, often called the null hypothesis, is that there is no agreement between the comparisons, and this is to be tested against the alternative hypothesis, H_1, that there is some agreement.

The n scores for each comparison are ranked, the rank r_{ij} denoting the rank of object j in comparison i, and all ranks lying between 1 and n. Average ranks are assigned to tied scores.

For each of the n objects, the k ranks are totalled, giving rank sums R_j, for $j = 1, 2, \ldots, n$. Under H_0, all the R_j would be approximately equal to the average rank sum $k(n+1)/2$. The total squared deviation of the R_j from this average value is therefore a measure of the departure from H_0 exhibited by the data. If there were complete agreement between the comparisons, the rank sums R_j would have the values $k, 2k, \ldots, nk$ (or some permutation thereof). The total squared deviation of these values is $k^2(n^3 - n)/12$.

Kendall's coefficient of concordance is the ratio

$$W = \frac{\sum_{j=1}^{n} \left(R_j - \frac{1}{2}k(n+1)\right)^2}{\frac{1}{12}k^2(n^3 - n)}$$

and lies between 0 and 1, the value 0 indicating complete disagreement, and 1 indicating complete agreement.

If there are tied rankings within comparisons, W is corrected by subtracting $k\sum T$ from the denominator, where $T = \sum(t^3 - t)/12$, each t being the number of occurrences of each tied rank within a comparison, and the summation of T being over all comparisons containing ties.

nag_nonpar_concordance_kendall (g08da) returns the value of W, and also an approximation, p, of the significance of the observed W. (For $n > 7, k(n-1)W$ approximately follows a χ^2_{n-1} distribution, so large values of W imply rejection of H_0.) H_0 is rejected by a test of chosen size α if $p < \alpha$. If $n \leq 7$, tables should be used to establish the significance of W (e.g., Table R of Siegel (1956)).

4 References

Siegel S (1956) *Non-parametric Statistics for the Behavioral Sciences* McGraw–Hill

5 Parameters

5.1 Compulsory Input Parameters

1: **x**$(ldx, \mathbf{n})$ – REAL (KIND=nag_wp) array

ldx, the first dimension of the array, must satisfy the constraint $ldx \geq \mathbf{k}$.

x(i, j) must be set to the value x_{ij} of object j in comparison i, for $i = 1, 2, \ldots, k$ and $j = 1, 2, \ldots, n$.

2: **k** – INTEGER

k, the number of comparisons.

Constraint: $\mathbf{k} \geq 2$.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the second dimension of the array **x**.

n, the number of objects.

Constraint: $\mathbf{n} \geq 2$.

5.3 Output Parameters

1: **w** – REAL (KIND=nag_wp)

The value of Kendall's coefficient of concordance, W.

2: **p** – REAL (KIND=nag_wp)

The approximate significance, p, of W.

3: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, $\mathbf{n} < 2$.

ifail $= 2$

On entry, $ldx < \mathbf{k}$.

ifail $= 3$

On entry, $\mathbf{k} \leq 1$.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

All computations are believed to be stable. The statistic W should be accurate enough for all practical uses.

8 Further Comments

The time taken by nag_nonpar_concordance_kendall (g08da) is approximately proportional to the product nk.

9 Example

This example is taken from page 234 of Siegel (1956). The data consists of 10 objects ranked on three different variables: x, y and z. The computed values of Kendall's coefficient is significant at the 1% level of significance $(p = 0.008 < 0.01)$, indicating that the null hypothesis of there being no agreement between the three rankings x, y, z may be rejected with reasonably high confidence.

9.1 Program Text

```
     function g08da_example

fprintf('g08da example results\n\n');

x = [1,   4.5, 2,   4.5, 3,   7.5, 6,  9,    7.5, 10;
     2.5, 1,   2.5, 4.5, 4.5, 8,   9,  6.5, 10,    6.5;
     2,   1,   4.5, 4.5, 4.5, 4.5, 8,  8,    8,   10  ];

fprintf('Kendall''s coefficient of concordance\n\n');
% Table Labels
labrow = 'Character';
rlabs  = {'Comparison 1 scores';
          'Comparison 2 scores';
          'Comparison 3 scores'};
labcol = 'None';
clabs  = {'       '};
ncols  = nag_int(80);
indent = nag_int(0);

[ifail] =  x04cb( ...
                'General', ' ', x, 'F5.1', 'Data values', labrow, ...
                rlabs, labcol, clabs, ncols, indent);

k = nag_int(3);
[w, p, ifail] = g08da(x, k);

fprintf('\nKendall''s coefficient = %8.3f\n', w);
fprintf('         Significance = %8.3f\n', p);
```

9.2 Program Results

```
     g08da example results

Kendall's coefficient of concordance

 Data values
 Comparison 1 scores   1.0  4.5  2.0  4.5  3.0  7.5  6.0  9.0  7.5 10.0
```

```
 Comparison 2 scores   2.5  1.0  2.5  4.5  4.5  8.0  9.0  6.5 10.0  6.5
 Comparison 3 scores   2.0  1.0  4.5  4.5  4.5  4.5  8.0  8.0  8.0 10.0

Kendall's coefficient =    0.828
         Significance =    0.008
```