NAG Toolbox

nag_rand_field_fracbm_generate (g05zt)

1 Purpose

nag_rand_field_fracbm_generate (g05zt) produces realizations of a fractional Brownian motion, using the circulant embedding method. The square roots of the eigenvalues of the extended covariance matrix (or embedding matrix) need to be input, and can be calculated using nag_rand_field_1d_predef_setup (g05zn).

2 Syntax

```
[state, z, xx, ifail] = nag_rand_field_fracbm_generate(ns, s, xmax, h, lam, rho,
state, 'm', m)
```

```
[state, z, xx, ifail] = g05zt(ns, s, xmax, h, lam, rho, state, 'm', m)
```

3 Description

The functions nag_rand_field_1d_predef_setup (g05zn) and nag_rand_field_fracbm_generate (g05zt) are used to simulate a fractional Brownian motion process with Hurst argument H over an interval $[0, x_{\max}]$, using a set of equally spaced points. Fractional Brownian motion itself cannot be simulated directly using this method, since it is not a stationary Gaussian random field; however its increments can be simulated like a stationary Gaussian random field. The circulant embedding method is described in the documentation for nag_rand_field_1d_predef_setup (g05zn).

nag_rand_field_fracbm_generate (g05zt) takes the square roots of the eigenvalues of the embedding matrix as returned by nag_rand_field_1d_predef_setup (g05zn) when **icov1** $= 14$, and its size M, as input and outputs S realizations of the fractional Brownian motion in Z.

One of the initialization functions nag_rand_init_repeat (g05kf) (for a repeatable sequence if computed sequentially) or nag_rand_init_nonrepeat (g05kg) (for a non-repeatable sequence) must be called prior to the first call to nag_rand_field_fracbm_generate (g05zt).

4 References

Dietrich C R and Newsam G N (1997) Fast and exact simulation of stationary Gaussian processes through circulant embedding of the covariance matrix *SIAM J. Sci. Comput.* **18** 1088–1107

Schlather M (1999) Introduction to positive definite functions and to unconditional simulation of random fields *Technical Report ST 99–10* Lancaster University

Wood A T A and Chan G (1994) Simulation of stationary Gaussian processes in $[0,1]^d$ *Journal of Computational and Graphical Statistics* **3(4)** 409–432

5 Parameters

5.1 Compulsory Input Parameters

1: **ns** – INTEGER

The number of steps (points) to be generated in realizations of the increments of the fractional Brownian motion. This must be the same value as supplied to nag_rand_field_1d_predef_setup (g05zn) when calculating the eigenvalues of the embedding matrix.

Note: in the context of fractional Brownian motion, **ns** represents the number of *steps* from a zero starting state. Realizations returned in **z** include this starting state and so $\mathbf{ns} + 1$ values are returned for each realization..

Constraint: $\mathbf{ns} \geq 1$.

2: **s** – INTEGER

S, the number of realizations of the fractional Brownian motion to simulate.

Constraint: $\mathbf{s} \geq 1$.

3: **xmax** – REAL (KIND=nag_wp)

The upper bound for the interval over which the fractional Brownian motion is to be simulated, as input to nag_rand_field_1d_user_setup (g05zm) or nag_rand_field_1d_predef_setup (g05zn).

Constraint: $\mathbf{xmax} > 0.0$.

4: **h** – REAL (KIND=nag_wp)

The Hurst parameter, H, for the fractional Brownian motion. This must be the same value as supplied to nag_rand_field_1d_predef_setup (g05zn) in **params**(1), when the eigenvalues of the embedding matrix were calculated.

Constraint: $0.0 < \mathbf{h} < 1.0$.

5: **lam**(**m**) – REAL (KIND=nag_wp) array

Contains the square roots of the eigenvalues of the embedding matrix, as returned by nag_rand_field_1d_user_setup (g05zm) or nag_rand_field_1d_predef_setup (g05zn).

Constraint: $\mathbf{lam}(i) \geq 0$, for $i = 1, 2, \ldots, \mathbf{m}$.

6: **rho** – REAL (KIND=nag_wp)

Indicates the scaling of the covariance matrix, as returned by nag_rand_field_1d_user_setup (g05zm) or nag_rand_field_1d_predef_setup (g05zn).

Constraint: $0.0 < \mathbf{rho} \leq 1.0$.

7: **state**(:) – INTEGER array

Note: the actual argument supplied **must** be the array **state** supplied to the initialization routines nag_rand_init_repeat (g05kf) or nag_rand_init_nonrepeat (g05kg).

Contains information on the selected base generator and its current state.

5.2 Optional Input Parameters

1: **m** – INTEGER

Default: the dimension of the array **lam**.

The size, M, of the embedding matrix, as returned by nag_rand_field_1d_user_setup (g05zm) or nag_rand_field_1d_predef_setup (g05zn).

Constraint: $\mathbf{m} \geq \max(1, 2(\mathbf{ns} - 1))$.

5.3 Output Parameters

1: **state**(:) – INTEGER array

Contains updated information on the state of the generator.

2: **z**(**ns** + **1**, **s**) – REAL (KIND=nag_wp) array

Contains the realizations of the fractional Brownian motion, Z. The jth realization, for the ith point **xx**(i), is stored in **z**(i, j), for $j = 1, 2, \ldots,$ **s** and $i = 1, 2, \ldots,$ **ns** $+ 1$.

3: **xx**(**ns** + **1**) – REAL (KIND=nag_wp) array

The points at which values of the fractional Brownian motion are output. The first point is always zero, and the subsequent **ns** points represent the equispaced steps towards the last point, **xmax**. Note that in nag_rand_field_1d_user_setup (g05zm) and nag_rand_field_1d_predef_setup (g05zn), the returned **ns** sample points are the mid-points of the grid returned in **xx** here.

4: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

Constraint: **ns** ≥ 1.

ifail $= 2$

Constraint: **s** ≥ 1.

ifail $= 3$

Constraint: **m** $\geq \max(1, 2($**ns** $- 1))$.

ifail $= 4$

Constraint: **xmax** > 0.0.

ifail $= 5$

Constraint: $0.0 <$ **h** < 1.0.

ifail $= 6$

On entry, at least one element of **lam** was negative.
Constraint: all elements of **lam** must be non-negative.

ifail $= 7$

Constraint: $0.0 <$ **rho** ≤ 1.0.

ifail $= 8$

On entry, **state** vector has been corrupted or not initialized.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

Not applicable.

8 Further Comments

None.

9 Example

This example calls nag_rand_field_fracbm_generate (g05zt) to generate 5 realizations of a fractional Brownian motion over 10 steps from $x = 0.0$ to $x = 2.0$ using eigenvalues generated by nag_rand_field_1d_predef_setup (g05zn) with **icov1** $= 14$.

9.1 Program Text

```
function g05zt_example

fprintf('g05zt example results\n\n');

% Upper bound for interval
xmax = 2;
% Number of sample points
ns   = nag_int(10);
% Scaling factor, rho = 1.
icorr = nag_int(2);

% Set fixed problem specifications for simulating fractional Brownian motion
h     = 0.35;
icov1 = nag_int(14);
np    = nag_int(2);
xmin  = 0;
var   = 1;
params = [h, xmax/double(ns)];

% Get square roots of the eigenvalues of the embedding matrix
[lam, xx, m, approx, rho, icount, eig, ifail] = ...
  g05zn( ...
         ns, xmin, xmax, var, icov1, params, ...
         'icorr', icorr, 'maxm', nag_int(2048));

fprintf('\nSize of embedding matrix = %d\n\n', m);

% Display approximation information if approximation used
if approx == 1
  fprintf('Approximation required\n\n');
  fprintf('rho = %10.5f\n', rho);
  fprintf('eig = %10.5f%10.5f%10.5f\n', eig(1:3));
  fprintf('icount = %d\n', icount);
else
  fprintf('Approximation not required\n\n');
end

% Initialize state array
genid = nag_int(1);
subid = nag_int(1);
seed  = [nag_int(14965)];
[state, ifail] = g05kf( ...
                        genid, subid, seed);

% Compute s fractional Brownian Motion realisations.
s    = nag_int(5);
[state, z, yy, ifail] = g05zt( ...
                               ns, s, xmax, h, lam(1:m), rho, state);

% Display random field results
% Set row labels to mesh points (column label is realisation number).
```

```
rlabs = cell(ns+1, 1);
for i=1:ns+1
  rlabs{i} = sprintf('%6.1f', yy(i));
end

% Matrix printing parameters
mtitle = 'Fractional Brownian motion realisations (x coordinate first):';
matrix = 'General';
diag   = 'Non-unit';
fmt    = 'f10.5';
rlabel = 'Character';
clabel = 'Integer';
clabs  = {' '};
ncols  = nag_int(80);
indent = nag_int(0);

[ifail] = x04cb( ...
                 matrix, diag, z, fmt, mtitle, rlabel, rlabs, clabel, ...
                 clabs, ncols, indent);
```

9.2 Program Results

```
     g05zt example results

Size of embedding matrix = 32

Approximation not required

 Fractional Brownian motion realisations (x coordinate first):
             1         2         3         4         5
 0.0   0.00000   0.00000   0.00000   0.00000   0.00000
 0.2  -0.52650  -0.16159  -0.96224  -0.40096   0.65803
 0.4  -1.81085  -0.85811  -1.43661   0.03947   0.99671
 0.6  -1.65690  -0.74802  -0.61733  -0.34685   0.05141
 0.8  -1.72240  -0.14958   0.14996   0.18134   0.26567
 1.0  -2.20349   0.46219   0.70982   0.66405   0.40706
 1.2  -2.38542   0.52085   0.36330   0.31831   0.81515
 1.4  -3.13939   0.68433   0.79826  -0.35408   1.12296
 1.6  -3.54602   0.64413   0.85751  -0.39303   1.14220
 1.8  -4.09082   1.67048   0.06038   0.30181   1.30350
 2.0  -2.97487   1.72275  -0.67253  -0.07439   1.57169
```