

NAG Toolbox

nag_rand_copula_gumbel (g05rk)

1 Purpose

nag_rand_copula_gumbel (g05rk) generates pseudorandom uniform variates with joint distribution of a Gumbel–Hougaard Archimedean copula.

2 Syntax

```
[state, x, ifail] = nag_rand_copula_gumbel(n, m, theta, sorder, state)
[state, x, ifail] = g05rk(n, m, theta, sorder, state)
```

3 Description

Generates n pseudorandom uniform m -variates whose joint distribution is the Gumbel–Hougaard Archimedean copula C_θ , given by

$$C_\theta = \exp\left\{-\left[(-\ln u_1)^\theta + (-\ln u_2)^\theta + \cdots + (-\ln u_m)^\theta\right]\right\}, \quad \begin{cases} \theta \in (1, \infty), \\ u_j \in (0, 1], \quad j = 1, 2, \dots, m; \end{cases}$$

with the special cases:

$C_1 = u_1 u_2 \cdots u_m$, the product copula;

$C_\infty = \min(u_1, u_2, \dots, u_m)$, the Fréchet–Hoeffding upper bound.

The generation method uses mixture of powers.

One of the initialization functions nag_rand_init_repeat (g05kf) (for a repeatable sequence if computed sequentially) or nag_rand_init_nonrepeat (g05kg) (for a non-repeatable sequence) must be called prior to the first call to nag_rand_copula_gumbel (g05rk).

4 References

Marshall A W and Olkin I (1988) Families of multivariate distributions *Journal of the American Statistical Association* **83** 403

Nelsen R B (2006) *An Introduction to Copulas* (2nd Edition) Springer Series in Statistics

5 Parameters

5.1 Compulsory Input Parameters

1: **n** – INTEGER

n , the number of pseudorandom uniform variates to generate.

Constraint: $n \geq 0$.

2: **m** – INTEGER

m , the number of dimensions.

Constraint: $m \geq 2$.

3: **theta** – REAL (KIND=nag_wp)

θ , the copula parameter.

Constraint: **theta** \geq 1.0.

4: **sorder** – INTEGER

Determines the storage order of variates; the (i, j) th variate is stored in $\mathbf{x}(i, j)$ if **sorder** = 1, and $\mathbf{x}(j, i)$ if **sorder** = 2, for $i = 1, 2, \dots, n$ and $j = 1, 2, \dots, m$.

Constraint: **sorder** = 1 or 2.

5: **state**(:) – INTEGER array

Note: the actual argument supplied **must** be the array **state** supplied to the initialization routines nag_rand_init_repeat (g05kf) or nag_rand_init_nonrepeat (g05kg).

Contains information on the selected base generator and its current state.

5.2 Optional Input Parameters

None.

5.3 Output Parameters

1: **state**(:) – INTEGER array

Contains updated information on the state of the generator.

2: $\mathbf{x}(ldx, sdx)$ – REAL (KIND=nag_wp) array

The pseudorandom uniform variates with joint distribution described by C_θ , with $\mathbf{x}(i, j)$ holding the i th value for the j th dimension if **sorder** = 1 and the j th value for the i th dimension of **sorder** = 2.

3: **ifail** – INTEGER

ifail = 0 unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = 1

On entry, corrupt **state** argument.

ifail = 2

Constraint: **theta** \geq 1.0.

ifail = 3

Constraint: **n** \geq 0.

ifail = 4

Constraint: **m** \geq 2.

ifail = 5

On entry, invalid **sorder**.

Constraint: **sorder** = 1 or 2.

ifail = 7

On entry, *ldx* is too small: *ldx* = *<value>*.

ifail = 8

On entry, *sdx* is too small: *sdx* = *<value>*.

ifail = -99

An unexpected error has been triggered by this routine. Please contact NAG.

ifail = -399

Your licence key may have expired or may not have been installed correctly.

ifail = -999

Dynamic memory allocation failed.

7 Accuracy

Not applicable.

8 Further Comments

In practice, the need for numerical stability restricts the range of θ such that:

if $(\theta - 1) < 1.0 \times 10^{-6}$, the function returns pseudorandom uniform variates with C_1 joint distribution;

if $\theta > \max(80.0, -0.5 \ln \epsilon_s)$, the function returns pseudorandom uniform variates with C_∞ joint distribution;

where ϵ_s is the safe-range parameter, the value of which is returned by `nag_machine_real_safe` (x02am).

9 Example

This example generates thirteen four-dimensional variates for copula $C_{2.4}$.

9.1 Program Text

```
function g05rk_example

fprintf('g05rk example results\n\n');

% Initialize the base generator to a repeatable sequence
seed = [nag_int(1762543)];
genid = nag_int(1);
subid = nag_int(1);
[state, ifail] = g05kf( ...
                    genid, subid, seed);

% Sample size
n = nag_int(13);
m = nag_int(4);
% Sample order
sorder = nag_int(1);

% Parameter
theta = 2.4;

% Generate variates
```

```
[state, x, ifail] = g05rk( ...  
                        n, m, theta, sorder, state);  
  
disp('Variates from a Gumbel--Hougaard copula');  
disp(x);
```

9.2 Program Results

g05rk example results

```
Variates from a Gumbel--Hougaard copula  
0.9369    0.8676    0.9713    0.8854  
0.1139    0.3063    0.8625    0.2743  
0.4418    0.2211    0.5042    0.4985  
0.7902    0.6007    0.7493    0.6474  
0.8362    0.9847    0.8807    0.9079  
0.1781    0.4610    0.1283    0.1329  
0.1272    0.1760    0.1805    0.0383  
0.4473    0.2171    0.1662    0.1300  
0.8899    0.9005    0.8844    0.8879  
0.9069    0.8681    0.8450    0.8804  
0.2222    0.5499    0.4965    0.6488  
0.3807    0.5967    0.5096    0.3577  
0.8445    0.7755    0.8661    0.8948
```
