

NAG Toolbox

nag_rand_copula_frank_bivar (g05rf)

1 Purpose

nag_rand_copula_frank_bivar (g05rf) generates pseudorandom uniform bivariate with joint distribution of a Frank Archimedean copula.

2 Syntax

```
[state, x, ifail] = nag_rand_copula_frank_bivar(n, theta, sorder, state)
[state, x, ifail] = g05rf(n, theta, sorder, state)
```

3 Description

Generates pseudorandom uniform bivariate $\{u_1, u_2\} \in [0, 1]^2$ whose joint distribution is the Frank Archimedean copula C_θ with parameter θ , given by

$$C_\theta = -\frac{1}{\theta} \ln \left[1 + \frac{(e^{-\theta u_1} - 1)(e^{-\theta u_2} - 1)}{e^{-\theta} - 1} \right], \quad \theta \in (-\infty, \infty) \setminus \{0\}$$

with the special cases:

$C_{-\infty} = \max(u_1 + u_2 - 1, 0)$, the Fréchet–Hoeffding lower bound;

$C_0 = u_1 u_2$, the product copula;

$C_\infty = \min(u_1, u_2)$, the Fréchet–Hoeffding upper bound.

The generation method uses conditional sampling.

One of the initialization functions nag_rand_init_repeat (g05kf) (for a repeatable sequence if computed sequentially) or nag_rand_init_nonrepeat (g05kg) (for a non-repeatable sequence) must be called prior to the first call to nag_rand_copula_frank_bivar (g05rf).

4 References

Nelsen R B (2006) *An Introduction to Copulas* (2nd Edition) Springer Series in Statistics

5 Parameters

5.1 Compulsory Input Parameters

1: **n** – INTEGER

n , the number of bivariate to generate.

Constraint: $n \geq 0$.

2: **theta** – REAL (KIND=nag_wp)

θ , the copula parameter.

3: **sorder** – INTEGER

Determines the storage order of variates; the (i, j) th variate is stored in $\mathbf{x}(i, j)$ if **sorder** = 1, and $\mathbf{x}(j, i)$ if **sorder** = 2, for $i = 1, 2, \dots, n$ and $j = 1, 2$.

Constraint: **sorder** = 1 or 2.

4: **state**(:) – INTEGER array

Note: the actual argument supplied **must** be the array **state** supplied to the initialization routines `nag_rand_init_repeat` (g05kf) or `nag_rand_init_nonrepeat` (g05kg).

Contains information on the selected base generator and its current state.

5.2 Optional Input Parameters

None.

5.3 Output Parameters

1: **state**(:) – INTEGER array

Contains updated information on the state of the generator.

2: **x**(*ldx*, *sdx*) – REAL (KIND=nag_wp) array

The n bivariate uniforms with joint distribution described by C_θ , with **x**(i, j) holding the i th value for the j th dimension if **sorder** = 1 and the j th value for the i th dimension if **sorder** = 2.

3: **ifail** – INTEGER

ifail = 0 unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = 1

On entry, corrupt **state** argument.

ifail = 3

Constraint: $n \geq 0$.

ifail = 4

On entry, invalid **sorder**.
Constraint: **sorder** = 1 or 2.

ifail = 6

On entry, *ldx* is too small: $ldx = \langle value \rangle$.

ifail = 7

On entry, *sdx* is too small: $sdx = \langle value \rangle$.

ifail = -99

An unexpected error has been triggered by this routine. Please contact NAG.

ifail = -399

Your licence key may have expired or may not have been installed correctly.

ifail = -999

Dynamic memory allocation failed.

7 Accuracy

Not applicable.

8 Further Comments

In practice, the need for numerical stability restricts the range of θ such that:

if $\theta < \ln \epsilon_s$, the function returns pseudorandom uniform variates with $C_{-\infty}$ joint distribution;

if $|\theta| < 1.0 \times 10^{-6}$, the function returns pseudorandom uniform variates with C_0 joint distribution;

if $\theta > \ln \epsilon$, the function returns pseudorandom uniform variates with C_{∞} joint distribution;

where ϵ_s is the safe-range parameter, the value of which is returned by `nag_machine_real_safe` (x02am); and ϵ is the *machine precision* returned by `nag_machine_precision` (x02aj).

9 Example

This example generates thirteen variates for copula $C_{-12.0}$.

9.1 Program Text

```
function g05rf_example

fprintf('g05rf example results\n\n');

% Initialize the base generator to a repeatable sequence
seed = [nag_int(1762543)];
genid = nag_int(1);
subid = nag_int(1);
[state, ifail] = g05kf( ...
                    genid, subid, seed);

% Sample size and order
n      = nag_int(13);
sorder = nag_int(1);

% Parameter
theta = -12;

% Generate variates
[state, x, ifail] = g05rf( ...
                        n, theta, sorder, state);

disp('Variates from a bivariate Frank copula');
disp(x);
```

9.2 Program Results

```
g05rf example results

Variates from a bivariate Frank copula
0.6364    0.1411
0.1065    0.8967
0.7460    0.1843
0.7983    0.1254
0.1046    0.9982
0.4925    0.6901
0.3843    0.6250
0.7871    0.1654
```

0.4982	0.5298
0.6717	0.2902
0.0505	0.9554
0.2580	0.8190
0.6238	0.3014
