NAG Toolbox

nag_stat_moments_quad_form (g01na)

1 Purpose

nag_stat_moments_quad_form (g01na) computes the cumulants and moments of quadratic forms in Normal variates.

2 Syntax

```
[rkum, rmom, ifail] = nag_stat_moments_quad_form(a, sigma, l, 'n', n, 'emu', emu)
[rkum, rmom, ifail] = g01na(a, sigma, l, 'n', n, 'emu', emu)
```

Note: the interface to this routine has changed since earlier releases of the toolbox:

At Mark 23: *mom* and *mean* were removed from the interface; **emu** was made optional.

3 Description

Let x have an n-dimensional multivariate Normal distribution with mean μ and variance-covariance matrix Σ. Then for a symmetric matrix A, nag_stat_moments_quad_form (g01na) computes up to the first 12 moments and cumulants of the quadratic form $Q = x^{\mathrm{T}}Ax$. The sth moment (about the origin) is defined as

$$E(Q^s),$$

where E denotes expectation. The sth moment of Q can also be found as the coefficient of $t^s/s!$ in the expansion of $E(e^{Qt})$. The sth cumulant is defined as the coefficient of $t^s/s!$ in the expansion of $\log\left(E(e^{Qt})\right)$.

The function is based on the function CUM written by Magnus and Pesaran (1993a) and based on the theory given by Magnus (1978), Magnus (1979) and Magnus (1986).

4 References

Magnus J R (1978) The moments of products of quadratic forms in Normal variables *Statist. Neerlandica* **32** 201–210

Magnus J R (1979) The expectation of products of quadratic forms in Normal variables: the practice *Statist. Neerlandica* **33** 131–136

Magnus J R (1986) The exact moments of a ratio of quadratic forms in Normal variables *Ann. Üconom. Statist.* **4** 95–109

Magnus J R and Pesaran B (1993a) The evaluation of cumulants and moments of quadratic forms in Normal variables (CUM): Technical description *Comput. Statist.* **8** 39–45

Magnus J R and Pesaran B (1993b) The evaluation of moments of quadratic forms and ratios of quadratic forms in Normal variables: Background, motivation and examples *Comput. Statist.* **8** 47–55

5 Parameters

5.1 Compulsory Input Parameters

1: $\mathbf{a}(lda, \mathbf{n})$ – REAL (KIND=nag_wp) array

lda, the first dimension of the array, must satisfy the constraint $lda \geq \mathbf{n}$.

The n by n symmetric matrix A. Only the lower triangle is referenced.

2: **sigma**($ldsig$, **n**) – REAL (KIND=nag_wp) array

$ldsig$, the first dimension of the array, must satisfy the constraint $ldsig \geq$ **n**.

The n by n variance-covariance matrix Σ. Only the lower triangle is referenced.

Constraint: the matrix Σ must be positive definite.

3: **l** – INTEGER

The required number of cumulants, and moments if specified.

Constraint: $1 \leq$ **l** ≤ 12.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the arrays **a**, **sigma** and the second dimension of the arrays **a**, **sigma**. (An error is raised if these dimensions are not equal.)

n, the dimension of the quadratic form.

Constraint: **n** > 1.

2: **emu**(:) – REAL (KIND=nag_wp) array

The dimension of the array **emu** must be at least **n** if $mean =$ 'M', and at least 1 otherwise

If supplied, **emu** must contain the n elements of the vector μ.

5.3 Output Parameters

1: **rkum**(**l**) – REAL (KIND=nag_wp) array

The **l** cumulants of the quadratic form.

2: **rmom**(:) – REAL (KIND=nag_wp) array

The dimension of the array **rmom** will be **l** if $mom =$ 'M' and 1 otherwise

If $mom =$ 'M', the **l** moments of the quadratic form.

3: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, **n** ≤ 1,
or **l** < 1,
or **l** > 12,
or $lda <$ **n**,
or $ldsig <$ **n**,
or $mom \neq$ 'C' or 'M',
or $mean \neq$ 'M' or 'Z'.

ifail $= 2$

On entry, the matrix Σ is not positive definite.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

In a range of tests the accuracy was found to be a modest multiple of ***machine precision***. See Magnus and Pesaran (1993b).

8 Further Comments

None.

9 Example

This example is given by Magnus and Pesaran (1993b) and considers the simple autoregression

$$y_t = \beta y_{t-1} + u_t, \quad t = 1, 2, \dots n,$$

where $\{u_t\}$ is a sequence of independent Normal variables with mean zero and variance one, and y_0 is known. The moments of the quadratic form

$$Q = \sum_{t=2}^{n} y_t y_{t-1}$$

are computed using nag_stat_moments_quad_form (g01na). The matrix A is given by:

$$\begin{aligned} A(i+1,i) &= \tfrac{1}{2}, \quad i = 1, 2, \dots n-1; \\ A(i,j) &= 0, \quad \text{otherwise.} \end{aligned}$$

The value of Σ can be computed using the relationships

$$\mathrm{var}\,(y_t) = \beta^2\,\mathrm{var}\,(y_{t-1}) + 1$$

and

$$\mathrm{cov}(y_t y_{t+k}) = \beta\,\mathrm{cov}(y_t y_{t+k-1})$$

for $k \geq 0$ and $\mathrm{var}(y_1) = 1$.

The values of β, y_0, n, and the number of moments required are read in and the moments and cumulants printed.

9.1 Program Text

```
function g01na_example

fprintf('g01na example results\n\n');

% Problem parameters
n    = 10;
l    = nag_int(4);
beta = 0.8;
con  = 1.0;

% Simple autoregression setup
a            = zeros(n,n);
```

```
a(2:n,1:n-1) = 0.5*eye(n-1);
emu = zeros(n,1);
for j=1:n
  emu(j) = con*beta^j;
end
sigma = zeros(n,n);
sigma(1,1) = 1;
for j = 2:n
  sigma(j,j) = sigma(j-1,j-1)*beta^2 + 1;
end
for i = 1:n
  s = sigma(i,i);
  for j = i+1:n
    sigma(j,i) = s*beta^(j-i);
  end
end

[rkum, rmom, ifail] = g01na( ...
                             a, sigma, l, 'emu', emu);

% Display results
fprintf(' n = %3d, beta = %6.3f, con = %6.3f\n\n', n, beta, con);
fprintf('    Cumulants       Moments\n\n');
ival = double([1:l]');
fprintf('%3d%12.4e    %12.4e\n',[ival rkum rmom]');
```

9.2 Program Results

```
     g01na example results

 n =  10, beta =  0.800, con =  1.000

    Cumulants       Moments

  1  1.7517e+01      1.7517e+01
  2  3.5010e+02      6.5695e+02
  3  1.6091e+04      3.9865e+04
  4  1.1700e+06      3.4039e+06
```