NAG Toolbox

nag_lapack_dstev (f08ja)

1 Purpose

nag_lapack_dstev (f08ja) computes all the eigenvalues and, optionally, all the eigenvectors of a real n by n symmetric tridiagonal matrix A.

2 Syntax

```
[d, e, z, info] = nag_lapack_dstev(jobz, d, e, 'n', n)
[d, e, z, info] = f08ja(jobz, d, e, 'n', n)
```

3 Description

nag_lapack_dstev (f08ja) computes all the eigenvalues and, optionally, all the eigenvectors of A using a combination of the QR and QL algorithms, with an implicit shift.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **jobz** – CHARACTER(1)

Indicates whether eigenvectors are computed.

jobz = 'N'
Only eigenvalues are computed.

jobz = 'V'
Eigenvalues and eigenvectors are computed.

Constraint: **jobz** = 'N' or 'V'.

2: **d**(:) – REAL (KIND=nag_wp) array

The dimension of the array **d** must be at least $\max(1, \mathbf{n})$

The n diagonal elements of the tridiagonal matrix A.

3: **e**(:) – REAL (KIND=nag_wp) array

The dimension of the array **e** must be at least $\max(1, \mathbf{n} - 1)$

The $(n - 1)$ subdiagonal elements of the tridiagonal matrix A.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the dimension of the array **d**.

n, the order of the matrix.

Constraint: $\mathbf{n} \geq 0$.

5.3 Output Parameters

1: **d**(:) – REAL (KIND=nag_wp) array

The dimension of the array **d** will be $\max(1, \mathbf{n})$

If $\mathbf{info} = 0$, the eigenvalues in ascending order.

2: **e**(:) – REAL (KIND=nag_wp) array

The dimension of the array **e** will be $\max(1, \mathbf{n} - 1)$

The contents of **e** are destroyed.

3: **z**(ldz, :) – REAL (KIND=nag_wp) array

The first dimension, ldz, of the array **z** will be

if $\mathbf{jobz} =$ 'V', $ldz = \max(1, \mathbf{n})$;
otherwise $ldz = 1$.

The second dimension of the array **z** will be $\max(1, \mathbf{n})$ if $\mathbf{jobz} =$ 'V' and 1 otherwise.

If $\mathbf{jobz} =$ 'V', then if $\mathbf{info} = 0$, **z** contains the orthonormal eigenvectors of the matrix A, with the ith column of Z holding the eigenvector associated with $\mathbf{d}(i)$.

If $\mathbf{jobz} =$ 'N', **z** is not referenced.

4: **info** – INTEGER

$\mathbf{info} = 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

$\mathbf{info} = -i$

If $\mathbf{info} = -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **jobz**, 2: **n**, 3: **d**, 4: **e**, 5: **z**, 6: **ldz**, 7: **work**, 8: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

$\mathbf{info} > 0$

If $\mathbf{info} = i$, the algorithm failed to converge; i off-diagonal elements of **e** did not converge to zero.

7 Accuracy

The computed eigenvalues and eigenvectors are exact for a nearby matrix $(A + E)$, where

$$\|E\|_2 = O(\epsilon)\|A\|_2,$$

and ϵ is the ***machine precision***. See Section 4.7 of Anderson *et al.* (1999) for further details.

8 Further Comments

The total number of floating-point operations is proportional to n^2 if **jobz** = 'N' and is proportional to n^3 if **jobz** = 'V'.

9 Example

This example finds all the eigenvalues and eigenvectors of the symmetric tridiagonal matrix

$$A = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 4 & 2 & 0 \\ 0 & 2 & 9 & 3 \\ 0 & 0 & 3 & 16 \end{pmatrix},$$

together with approximate error bounds for the computed eigenvalues and eigenvectors.

9.1 Program Text

```
     function f08ja_example

fprintf('f08ja example results\n\n');

% Symmetric tridiagonal matrix A stored as main and sub-diagonals
n = nag_int(4);
d = [1;     4;     9;     16];
e = [1;     2;     3];

% All eigenvalues and eigenvectors of A
jobz = 'Vectors';
[w, ~, z, info] = f08ja( ...
                         jobz, d, e);

% Normalize eigenvectors: largest element positive
for j = 1:n
  [~,k] = max(abs(z(:,j)));
  if z(k,j) < 0;
    z(:,j) = -z(:,j);
  end
end

disp('Eigenvalues');
disp(w);
disp('Eigenvectors');
disp(z);

% Eigenvalue error bound
errbnd = x02aj*max(abs(w(1)),abs(w(end)));
% Eigenvector condition numbers
[rcondz, info] = f08fl( ...
         'Eigenvectors', n, n, w);

% Eigenvector error bounds
zerrbd = errbnd./rcondz;

disp('Error estimate for the eigenvalues');
fprintf('%12.1e\n',errbnd);
disp('Error estimates for the eigenvectors');
fprintf('%12.1e',zerrbd);
fprintf('\n');
```

9.2 Program Results

```
     f08ja example results

Eigenvalues
    0.6476
    3.5470
    8.6578
   17.1477

Eigenvectors
    0.9396    0.3388    0.0494    0.0034
   -0.3311    0.8628    0.3781    0.0545
    0.0853   -0.3648    0.8558    0.3568
   -0.0167    0.0879   -0.3497    0.9326

Error estimate for the eigenvalues
     1.9e-15
Error estimates for the eigenvectors
     6.6e-16     6.6e-16     3.7e-16     2.2e-16
```