NAG Toolbox

nag_lapack_dorgql (f08cf)

1 Purpose

nag_lapack_dorgql (f08cf) generates all or part of the real m by m orthogonal matrix Q from a QL factorization computed by nag_lapack_dgeqlf (f08ce).

2 Syntax

```
[a, info] = nag_lapack_dorgql(a, tau, 'm', m, 'n', n, 'k', k)
[a, info] = f08cf(a, tau, 'm', m, 'n', n, 'k', k)
```

3 Description

nag_lapack_dorgql (f08cf) is intended to be used after a call to nag_lapack_dgeqlf (f08ce), which performs a QL factorization of a real matrix A. The orthogonal matrix Q is represented as a product of elementary reflectors.

This function may be used to generate Q explicitly as a square matrix, or to form only its trailing columns.

Usually Q is determined from the QL factorization of an m by p matrix A with $m \geq p$. The whole of Q may be computed by:

```
[a, info] = f08cf(a, tau);
```

(note that the array **a** must have at least m columns) or its trailing p columns by:

```
[a, info] = f08cf(a(:,1:p), tau);
```

The columns of Q returned by the last call form an orthonormal basis for the space spanned by the columns of A; thus nag_lapack_dgeqlf (f08ce) followed by nag_lapack_dorgql (f08cf) can be used to orthogonalize the columns of A.

The information returned by nag_lapack_dgeqlf (f08ce) also yields the QL factorization of the trailing k columns of A, where $k < p$. The orthogonal matrix arising from this factorization can be computed by:

```
[a, info] = f08cf(a, tau, 'k', k);
```

or its trailing k columns by:

```
[a, info] = f08cf(a(:,1:p), tau, 'k', k);
```

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **a**$(lda,:)$ – REAL (KIND=nag_wp) array

The first dimension of the array **a** must be at least $\max(1,\mathbf{m})$.

The second dimension of the array **a** must be at least $\max(1, \mathbf{n})$.

Details of the vectors which define the elementary reflectors, as returned by nag_lapack_dgeqlf (f08ce).

2: **tau**(:) – REAL (KIND=nag_wp) array

The dimension of the array **tau** must be at least $\max(1, \mathbf{k})$

Further details of the elementary reflectors, as returned by nag_lapack_dgeqlf (f08ce).

5.2 Optional Input Parameters

1: **m** – INTEGER

Default: the first dimension of the array **a**.

m, the number of rows of the matrix Q.

Constraint: $\mathbf{m} \geq 0$.

2: **n** – INTEGER

Default: the second dimension of the array **a**.

n, the number of columns of the matrix Q.

Constraint: $\mathbf{m} \geq \mathbf{n} \geq 0$.

3: **k** – INTEGER

Default: the dimension of the array **tau**.

k, the number of elementary reflectors whose product defines the matrix Q.

Constraint: $\mathbf{n} \geq \mathbf{k} \geq 0$.

5.3 Output Parameters

1: **a**(lda, :) – REAL (KIND=nag_wp) array

The first dimension of the array **a** will be $\max(1, \mathbf{m})$.

The second dimension of the array **a** will be $\max(1, \mathbf{n})$.

The m by n matrix Q.

2: **info** – INTEGER

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **m**, 2: **n**, 3: **k**, 4: **a**, 5: **lda**, 6: **tau**, 7: **work**, 8: **lwork**, 9: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The computed matrix Q differs from an exactly orthogonal matrix by a matrix E such that

$$\|E\|_2 = O(\epsilon),$$

where ϵ is the ***machine precision***.

8 Further Comments

The total number of floating-point operations is approximately $4mnk - 2(m+n)k^2 + \frac{4}{3}k^3$; when $n = k$, the number is approximately $\frac{2}{3}n^2(3m - n)$.

The complex analogue of this function is nag_lapack_zungql (f08ct).

9 Example

This example generates the first four columns of the matrix Q of the QL factorization of A as returned by nag_lapack_dgeqlf (f08ce), where

$$A = \begin{pmatrix} -0.57 & -1.28 & -0.39 & 0.25 \\ -1.93 & 1.08 & -0.31 & -2.14 \\ 2.30 & 0.24 & 0.40 & -0.35 \\ -1.93 & 0.64 & -0.66 & 0.08 \\ 0.15 & 0.30 & 0.15 & -2.13 \\ -0.02 & 1.03 & -1.43 & 0.50 \end{pmatrix}.$$

Note that the block size (NB) of 64 assumed in this example is not realistic for such a small problem, but should be suitable for large problems.

9.1 Program Text

```
     function f08cf_example

fprintf('f08cf example results\n\n');

% Form Q from QL factorization of A
m = 6;
n = 4;
a = [-0.57, -1.28, -0.39,  0.25;
     -1.93,  1.08, -0.31, -2.14;
      2.30,  0.24,  0.40, -0.35;
     -1.93,  0.64, -0.66,  0.08;
      0.15,  0.3,   0.15, -2.13;
     -0.02,  1.03, -1.43,  0.5];

% Compute the QL factorization of A
[ql, tau, info] = f08ce(a);

% Form Q
[Q, info] = f08cf(ql, tau);
% Print Q
[ifail] = x04ca( ...
  'General', ' ', Q, 'Orthogonal Q from QL of A');
```

9.2 Program Results

```
    f08cf example results

 Orthogonal Q from QL of A
          1       2       3       4
 1  -0.0833  0.9100 -0.2202 -0.0809
 2   0.2972 -0.1080 -0.2706  0.6922
 3  -0.6404 -0.2351  0.2220  0.1132
 4   0.4461 -0.1620 -0.3866 -0.0259
 5  -0.2938  0.2022  0.0015  0.6890
 6  -0.4575 -0.1946 -0.8243 -0.1617
```