

NAG Toolbox

nag_lapack_dtbcon (f07vg)

1 Purpose

nag_lapack_dtbcon (f07vg) estimates the condition number of a real triangular band matrix.

2 Syntax

```
[rcond, info] = nag_lapack_dtbcon(norm_p, uplo, diag, kd, ab, 'n', n)
[rcond, info] = f07vg(norm_p, uplo, diag, kd, ab, 'n', n)
```

3 Description

nag_lapack_dtbcon (f07vg) estimates the condition number of a real triangular band matrix A , in either the 1-norm or the ∞ -norm:

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1 \quad \text{or} \quad \kappa_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty.$$

Note that $\kappa_\infty(A) = \kappa_1(A^T)$.

Because the condition number is infinite if A is singular, the function actually returns an estimate of the **reciprocal** of the condition number.

The function computes $\|A\|_1$ or $\|A\|_\infty$ exactly, and uses Higham's implementation of Hager's method (see Higham (1988)) to estimate $\|A^{-1}\|_1$ or $\|A^{-1}\|_\infty$.

4 References

Higham N J (1988) FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

5 Parameters

5.1 Compulsory Input Parameters

1: **norm_p** – CHARACTER(1)

Indicates whether $\kappa_1(A)$ or $\kappa_\infty(A)$ is estimated.

norm_p = '1' or 'O'

$\kappa_1(A)$ is estimated.

norm_p = 'I'

$\kappa_\infty(A)$ is estimated.

Constraint: **norm_p** = '1', 'O' or 'I'.

2: **uplo** – CHARACTER(1)

Specifies whether A is upper or lower triangular.

uplo = 'U'

A is upper triangular.

uplo = 'L'

A is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

3: **diag** – CHARACTER(1)

Indicates whether A is a nonunit or unit triangular matrix.

diag = 'N'

A is a nonunit triangular matrix.

diag = 'U'

A is a unit triangular matrix; the diagonal elements are not referenced and are assumed to be 1.

Constraint: **diag** = 'N' or 'U'.

4: **kd** – INTEGER

k_d , the number of superdiagonals of the matrix A if **uplo** = 'U', or the number of subdiagonals if **uplo** = 'L'.

Constraint: **kd** \geq 0.

5: **ab**(*ldab*,:) – REAL (KIND=nag_wp) array

The first dimension of the array **ab** must be at least **kd** + 1.

The second dimension of the array **ab** must be at least $\max(1, \mathbf{n})$.

The n by n triangular band matrix A .

The matrix is stored in rows 1 to $k_d + 1$, more precisely,

if **uplo** = 'U', the elements of the upper triangle of A within the band must be stored with element A_{ij} in **ab**($k_d + 1 + i - j, j$) for $\max(1, j - k_d) \leq i \leq j$;

if **uplo** = 'L', the elements of the lower triangle of A within the band must be stored with element A_{ij} in **ab**($1 + i - j, j$) for $j \leq i \leq \min(n, j + k_d)$.

If **diag** = 'U', the diagonal elements of A are assumed to be 1, and are not referenced.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the second dimension of the array **ab**.

n , the order of the matrix A .

Constraint: **n** \geq 0.

5.3 Output Parameters

1: **rcond** – REAL (KIND=nag_wp)

An estimate of the reciprocal of the condition number of A . **rcond** is set to zero if exact singularity is detected or the estimate underflows. If **rcond** is less than *machine precision*, A is singular to working precision.

2: **info** – INTEGER

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info < 0

If **info** = $-i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

The computed estimate **rcond** is never less than the true value ρ , and in practice is nearly always less than 10ρ , although examples can be constructed where **rcond** is much larger.

8 Further Comments

A call to `nag_lapack_dtbcon` (f07vg) involves solving a number of systems of linear equations of the form $Ax = b$ or $A^T x = b$; the number is usually 4 or 5 and never more than 11. Each solution involves approximately $2nk$ floating-point operations (assuming $n \gg k$) but takes considerably longer than a call to `nag_lapack_dtbtrs` (f07ve) with one right-hand side, because extra care is taken to avoid overflow when A is approximately singular.

The complex analogue of this function is `nag_lapack_ztbcon` (f07vu).

9 Example

This example estimates the condition number in the 1-norm of the matrix A , where

$$A = \begin{pmatrix} -4.16 & 0.00 & 0.00 & 0.00 \\ -2.25 & 4.78 & 0.00 & 0.00 \\ 0.00 & 5.86 & 6.32 & 0.00 \\ 0.00 & 0.00 & -4.82 & 0.16 \end{pmatrix}.$$

Here A is treated as a lower triangular band matrix with one subdiagonal. The true condition number in the 1-norm is 69.62.

9.1 Program Text

```
function f07vg_example

fprintf('f07vg example results\n\n');

% Condition number of A, where A is lower triangular banded
% and stored in triangular/symmetric banded format
kd = nag_int(1);
ab = [-4.16, 4.78, 6.32, 0.16;
      -2.25, 5.86, -4.82, 0.00];

% Reciprocal condition number
norm_p = '1';
uplo = 'L';
diag = 'N';
[rcond, info] = f07vg( ...
                 norm_p, uplo, diag, kd, ab);

fprintf('Estimate of condition number = %9.2e\n', 1/rcond);
```

9.2 Program Results

```
f07vg example results

Estimate of condition number = 6.96e+01
```
