NAG Toolbox

nag_lapack_dtbtrs (f07ve)

1 Purpose

nag_lapack_dtbtrs (f07ve) solves a real triangular band system of linear equations with multiple right-hand sides, $AX = B$ or $A^{\mathrm{T}}X = B$.

2 Syntax

```
[b, info] = nag_lapack_dtbtrs(uplo, trans, diag, kd, ab, b, 'n', n, 'nrhs_p',
nrhs_p)
```

```
[b, info] = f07ve(uplo, trans, diag, kd, ab, b, 'n', n, 'nrhs_p', nrhs_p)
```

3 Description

nag_lapack_dtbtrs (f07ve) solves a real triangular band system of linear equations $AX = B$ or $A^{\mathrm{T}}X = B$.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Higham N J (1989) The accuracy of solutions to triangular systems *SIAM J. Numer. Anal.* **26** 1252–1265

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – CHARACTER(1)

Specifies whether A is upper or lower triangular.

uplo = 'U'
A is upper triangular.

uplo = 'L'
A is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **trans** – CHARACTER(1)

Indicates the form of the equations.

trans = 'N'
The equations are of the form $AX = B$.

trans = 'T' or 'C'
The equations are of the form $A^{\mathrm{T}}X = B$.

Constraint: **trans** = 'N', 'T' or 'C'.

3: **diag** – CHARACTER(1)

Indicates whether A is a nonunit or unit triangular matrix.

diag = 'N'
A is a nonunit triangular matrix.

diag = 'U'
A is a unit triangular matrix; the diagonal elements are not referenced and are assumed to be 1.

Constraint: **diag** = 'N' or 'U'.

4: **kd** – INTEGER

k_d, the number of superdiagonals of the matrix A if **uplo** = 'U', or the number of subdiagonals if **uplo** = 'L'.

Constraint: **kd** ≥ 0.

5: **ab**$(ldab,:)$ – REAL (KIND=nag_wp) array

The first dimension of the array **ab** must be at least **kd** $+ 1$.

The second dimension of the array **ab** must be at least $\max(1, \mathbf{n})$.

The n by n triangular band matrix A.

The matrix is stored in rows 1 to $k_d + 1$, more precisely,

if **uplo** = 'U', the elements of the upper triangle of A within the band must be stored with element A_{ij} in $\mathbf{ab}(k_d + 1 + i - j, j)$ for $\max(1, j - k_d) \leq i \leq j$;

if **uplo** = 'L', the elements of the lower triangle of A within the band must be stored with element A_{ij} in $\mathbf{ab}(1 + i - j, j)$ for $j \leq i \leq \min(n, j + k_d)$.

If **diag** = 'U', the diagonal elements of A are assumed to be 1, and are not referenced.

6: **b**$(ldb,:)$ – REAL (KIND=nag_wp) array

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$.

The second dimension of the array **b** must be at least $\max(1, \mathbf{nrhs\_p})$.

The n by r right-hand side matrix B.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the second dimension of the array **ab**.

n, the order of the matrix A.

Constraint: **n** ≥ 0.

2: **nrhs_p** – INTEGER

Default: the second dimension of the array **b**.

r, the number of right-hand sides.

Constraint: **nrhs_p** ≥ 0.

5.3 Output Parameters

1: **b**$(ldb,:)$ – REAL (KIND=nag_wp) array

The first dimension of the array **b** will be $\max(1, \mathbf{n})$.

The second dimension of the array **b** will be $\max(1, \mathbf{nrhs\_p})$.

The n by r solution matrix X.

2: **info** – INTEGER

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info < 0

If **info** $= -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

info > 0 (*warning*)

Element $\langle value \rangle$ of the diagonal is exactly zero. A is singular and the solution has not been computed.

7 Accuracy

The solutions of triangular systems of equations are usually computed to high accuracy. See Higham (1989).

For each right-hand side vector b, the computed solution x is the exact solution of a perturbed system of equations $(A+E)x = b$, where

$$|E| \leq c(k)\epsilon|A|,$$

$c(k)$ is a modest linear function of k, and ϵ is the ***machine precision***.

If $\hat{x}$ is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x-\hat{x}\|_\infty}{\|x\|_\infty} \leq c(k)\,\mathrm{cond}(A,x)\epsilon, \qquad \text{provided} \qquad c(k)\,\mathrm{cond}(A,x)\epsilon < 1,$$

where $\mathrm{cond}(A,x) = \left\| |A^{-1}||A||x| \right\|_\infty / \|x\|_\infty$.

Note that $\mathrm{cond}(A,x) \leq \mathrm{cond}(A) = \left\| |A^{-1}||A| \right\|_\infty \leq \kappa_\infty(A)$; $\mathrm{cond}(A,x)$ can be much smaller than $\mathrm{cond}(A)$ and it is also possible for $\mathrm{cond}(A^{\mathrm{T}})$ to be much larger (or smaller) than $\mathrm{cond}(A)$.

Forward and backward error bounds can be computed by calling nag_lapack_dtbrfs (f07vh), and an estimate for $\kappa_\infty(A)$ can be obtained by calling nag_lapack_dtbcon (f07vg) with **norm_p** = 'I'.

8 Further Comments

The total number of floating-point operations is approximately $2nkr$ if $k \ll n$.

The complex analogue of this function is nag_lapack_ztbtrs (f07vs).

9 Example

This example solves the system of equations $AX = B$, where

$$A = \begin{pmatrix} -4.16 & 0.00 & 0.00 & 0.00 \\ -2.25 & 4.78 & 0.00 & 0.00 \\ 0.00 & 5.86 & 6.32 & 0.00 \\ 0.00 & 0.00 & -4.82 & 0.16 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -16.64 & -4.16 \\ -13.78 & -16.59 \\ 13.10 & -4.94 \\ -14.14 & -9.96 \end{pmatrix}.$$

Here A is treated as a lower triangular band matrix with one subdiagonal.

9.1 Program Text

```
function f07ve_example

fprintf('f07ve example results\n\n');

% Solve AX=B, where A is lower triangular banded
% and stored in triangular/symmetric banded format
kd = nag_int(1);
ab = [-4.16, 4.78,  6.32, 0.16;
      -2.25, 5.86, -4.82, 0.00];
b = [-16.64,  -4.16;
     -13.78, -16.59;
      13.10,  -4.94;
     -14.14,  -9.96];

% Solve
uplo  = 'L';
trans = 'N';
diag  = 'N';
[x, info] = f07ve( ...
                   uplo, trans, diag, kd, ab, b);

% Display solution
[ifail] = x04ca( ...
                 'Gen', diag, x, 'Solution(s)');
```

9.2 Program Results

```
f07ve example results

 Solution(s)
             1          2
 1      4.0000     1.0000
 2     -1.0000    -3.0000
 3      3.0000     2.0000
 4      2.0000    -2.0000
```