NAG Toolbox

nag_lapack_dtrtrs (f07te)

1 Purpose

nag_lapack_dtrtrs (f07te) solves a real triangular system of linear equations with multiple right-hand sides, $AX = B$ or $A^{\mathrm{T}}X = B$.

2 Syntax

```
[b, info] = nag_lapack_dtrtrs(uplo, trans, diag, a, b, 'n', n, 'nrhs_p', nrhs_p)
[b, info] = f07te(uplo, trans, diag, a, b, 'n', n, 'nrhs_p', nrhs_p)
```

3 Description

nag_lapack_dtrtrs (f07te) solves a real triangular system of linear equations $AX = B$ or $A^{\mathrm{T}}X = B$.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Higham N J (1989) The accuracy of solutions to triangular systems *SIAM J. Numer. Anal.* **26** 1252–1265

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – CHARACTER(1)

Specifies whether A is upper or lower triangular.

uplo = 'U'
A is upper triangular.

uplo = 'L'
A is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **trans** – CHARACTER(1)

Indicates the form of the equations.

trans = 'N'
The equations are of the form $AX = B$.

trans = 'T' or 'C'
The equations are of the form $A^{\mathrm{T}}X = B$.

Constraint: **trans** = 'N', 'T' or 'C'.

3: **diag** – CHARACTER(1)

Indicates whether A is a nonunit or unit triangular matrix.

diag = 'N'
A is a nonunit triangular matrix.

diag = 'U'
A is a unit triangular matrix; the diagonal elements are not referenced and are assumed to be 1.

Constraint: **diag** = 'N' or 'U'.

4: **a**(lda, :) – REAL (KIND=nag_wp) array

The first dimension of the array **a** must be at least max(1, **n**).

The second dimension of the array **a** must be at least max(1, **n**).

The n by n triangular matrix A.

If **uplo** = 'U', a is upper triangular and the elements of the array below the diagonal are not referenced.

If **uplo** = 'L', a is lower triangular and the elements of the array above the diagonal are not referenced.

If **diag** = 'U', the diagonal elements of a are assumed to be 1, and are not referenced.

5: **b**(ldb, :) – REAL (KIND=nag_wp) array

The first dimension of the array **b** must be at least max(1, **n**).

The second dimension of the array **b** must be at least max(1, **nrhs_p**).

The n by r right-hand side matrix B.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the arrays **a**, **b** and the second dimension of the array **a**.

n, the order of the matrix A.

Constraint: **n** ≥ 0.

2: **nrhs_p** – INTEGER

Default: the second dimension of the array **b**.

r, the number of right-hand sides.

Constraint: **nrhs_p** ≥ 0.

5.3 Output Parameters

1: **b**(ldb, :) – REAL (KIND=nag_wp) array

The first dimension of the array **b** will be max(1, **n**).

The second dimension of the array **b** will be max(1, **nrhs_p**).

The n by r solution matrix X.

2: **info** – INTEGER

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info < 0

If **info** = $-i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

info > 0 (*warning*)

Element $\langle value \rangle$ of the diagonal is exactly zero. A is singular and the solution has not been computed.

7 Accuracy

The solutions of triangular systems of equations are usually computed to high accuracy. See Higham (1989).

For each right-hand side vector b, the computed solution x is the exact solution of a perturbed system of equations $(A + E)x = b$, where

$$|E| \leq c(n)\epsilon|A|,$$

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

If $\hat{x}$ is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x - \hat{x}\|_\infty}{\|x\|_\infty} \leq c(n)\,\mathrm{cond}(A, x)\epsilon, \qquad \text{provided} \qquad c(n)\,\mathrm{cond}(A, x)\epsilon < 1,$$

where $\mathrm{cond}(A, x) = \left\| |A^{-1}||A||x| \right\|_\infty / \|x\|_\infty$.

Note that $\mathrm{cond}(A, x) \leq \mathrm{cond}(A) = \left\| |A^{-1}||A| \right\|_\infty \leq \kappa_\infty(A)$; $\mathrm{cond}(A, x)$ can be much smaller than $\mathrm{cond}(A)$ and it is also possible for $\mathrm{cond}(A^{\mathrm{T}})$ to be much larger (or smaller) than $\mathrm{cond}(A)$.

Forward and backward error bounds can be computed by calling nag_lapack_dtrrfs (f07th), and an estimate for $\kappa_\infty(A)$ can be obtained by calling nag_lapack_dtrcon (f07tg) with **norm_p** = 'I'.

8 Further Comments

The total number of floating-point operations is approximately n^2r.

The complex analogue of this function is nag_lapack_ztrtrs (f07ts).

9 Example

This example solves the system of equations $AX = B$, where

$$A = \begin{pmatrix} 4.30 & 0.00 & 0.00 & 0.00 \\ -3.96 & -4.87 & 0.00 & 0.00 \\ 0.40 & 0.31 & -8.02 & 0.00 \\ -0.27 & 0.07 & -5.95 & 0.12 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -12.90 & -21.50 \\ 16.75 & 14.93 \\ -17.55 & 6.33 \\ -11.04 & 8.09 \end{pmatrix}.$$

9.1 Program Text

```
     function f07te_example

fprintf('f07te example results\n\n');

% Solve AX=B where A is Lower triangular
a = [ 4.30,  0,      0,     0;
     -3.96, -4.87,  0,     0;
      0.40,  0.31, -8.02, 0;
     -0.27,  0.07, -5.95, 0.12];
b = [-12.90, -21.50;
      16.75,  14.93;
     -17.55,   6.33;
     -11.04,   8.09];

uplo = 'L';
trans = 'N';
diag = 'N';

% Solve
```

```
[x, info] = f07te( ...
                   uplo, trans, diag, a, b);

% Display solution
[ifail] = x04ca( ...
                 'Gen', diag, x, 'Solution(s)');
```

9.2 Program Results

```
     f07te example results

 Solution(s)
             1          2
 1     -3.0000    -5.0000
 2     -1.0000     1.0000
 3      2.0000    -1.0000
 4      1.0000     6.0000
```