NAG Toolbox

nag_lapack_zsptrs (f07qs)

1 Purpose

nag_lapack_zsptrs (f07qs) solves a complex symmetric system of linear equations with multiple right-hand sides,

$$AX = B,$$

where A has been factorized by nag_lapack_zsptrf (f07qr), using packed storage.

2 Syntax

```
[b, info] = nag_lapack_zsptrs(uplo, ap, ipiv, b, 'n', n, 'nrhs_p', nrhs_p)
[b, info] = f07qs(uplo, ap, ipiv, b, 'n', n, 'nrhs_p', nrhs_p)
```

3 Description

nag_lapack_zsptrs (f07qs) is used to solve a complex symmetric system of linear equations $AX = B$, the function must be preceded by a call to nag_lapack_zsptrf (f07qr) which computes the Bunch–Kaufman factorization of A, using packed storage.

If **uplo** = 'U', $A = PUDU^{T}P^{T}$, where P is a permutation matrix, U is an upper triangular matrix and D is a symmetric block diagonal matrix with 1 by 1 and 2 by 2 blocks; the solution X is computed by solving $PUDY = B$ and then $U^{T}P^{T}X = Y$.

If **uplo** = 'L', $A = PLDL^{T}P^{T}$, where L is a lower triangular matrix; the solution X is computed by solving $PLDY = B$ and then $L^{T}P^{T}X = Y$.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – CHARACTER(1)

Specifies how A has been factorized.

uplo = 'U'
$A = PUDU^{T}P^{T}$, where U is upper triangular.

uplo = 'L'
$A = PLDL^{T}P^{T}$, where L is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **ap**(:) – COMPLEX (KIND=nag_wp) array

The dimension of the array **ap** must be at least $\max(1, \mathbf{n} \times (\mathbf{n}+1)/2)$

The factorization of A stored in packed form, as returned by nag_lapack_zsptrf (f07qr).

3: **ipiv**(:) – INTEGER array

The dimension of the array **ipiv** must be at least $\max(1, \mathbf{n})$

Details of the interchanges and the block structure of D, as returned by nag_lapack_zsptrf (f07qr).

4: **b**(ldb, :) – COMPLEX (KIND=nag_wp) array

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$.

The second dimension of the array **b** must be at least $\max(1, \mathbf{nrhs\_p})$.

The n by r right-hand side matrix B.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the array **ap** and the second dimension of the array **ap**. (An error is raised if these dimensions are not equal.)

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

2: **nrhs_p** – INTEGER

Default: the second dimension of the array **b**.

r, the number of right-hand sides.

Constraint: $\mathbf{nrhs\_p} \geq 0$.

5.3 Output Parameters

1: **b**(ldb, :) – COMPLEX (KIND=nag_wp) array

The first dimension of the array **b** will be $\max(1, \mathbf{n})$.

The second dimension of the array **b** will be $\max(1, \mathbf{nrhs\_p})$.

The n by r solution matrix X.

2: **info** – INTEGER

$\mathbf{info} = 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

$\mathbf{info} < 0$

If $\mathbf{info} = -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

For each right-hand side vector b, the computed solution x is the exact solution of a perturbed system of equations $(A + E)x = b$, where

if **uplo** = 'U', $|E| \leq c(n)\epsilon P|U||D||U^{\mathrm{T}}|P^{\mathrm{T}}$;

if **uplo** = 'L', $|E| \leq c(n)\epsilon P|L||D||L^{\mathrm{T}}|P^{\mathrm{T}}$,

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

If $\hat{x}$ is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x - \hat{x}\|_\infty}{\|x\|_\infty} \leq c(n)\,\mathrm{cond}(A, x)\epsilon$$

where $\mathrm{cond}(A, x) = \left\| |A^{-1}||A||x| \right\|_\infty / \|x\|_\infty \leq \mathrm{cond}(A) = \left\| |A^{-1}||A| \right\|_\infty \leq \kappa_\infty(A)$.

Note that $\mathrm{cond}(A, x)$ can be much smaller than $\mathrm{cond}(A)$.

Forward and backward error bounds can be computed by calling nag_lapack_zsprfs (f07qv), and an estimate for $\kappa_\infty(A)$ ($= \kappa_1(A)$) can be obtained by calling nag_lapack_zspcon (f07qu).

8 Further Comments

The total number of real floating-point operations is approximately $8n^2r$.

This function may be followed by a call to nag_lapack_zsprfs (f07qv) to refine the solution and return an error estimate.

The real analogue of this function is nag_lapack_dsptrs (f07pe).

9 Example

This example solves the system of equations $AX = B$, where

$$A = \begin{pmatrix} -0.39 - 0.71i & 5.14 - 0.64i & -7.86 - 2.96i & 3.80 + 0.92i \\ 5.14 - 0.64i & 8.86 + 1.81i & -3.52 + 0.58i & 5.32 - 1.59i \\ -7.86 - 2.96i & -3.52 + 0.58i & -2.83 - 0.03i & -1.54 - 2.86i \\ 3.80 + 0.92i & 5.32 - 1.59i & -1.54 - 2.86i & -0.56 + 0.12i \end{pmatrix}$$

and

$$B = \begin{pmatrix} -55.64 + 41.22i & -19.09 - 35.97i \\ -48.18 + 66.00i & -12.08 - 27.02i \\ -0.49 - \ \ 1.47i & 6.95 + 20.49i \\ -6.43 + 19.24i & -4.59 - 35.53i \end{pmatrix}.$$

Here A is symmetric, stored in packed form, and must first be factorized by nag_lapack_zsptrf (f07qr).

9.1 Program Text

```
     function f07qs_example

fprintf('f07qs example results\n\n');

% Solve Ax = B, where A is complex symmetric matrix such that the
% lower triangular part is stored in packed format
uplo = 'L';
n = nag_int(4);
ap = [ -0.39 -  0.71i,   5.14 -  0.64i,  -7.86 - 2.96i,   3.80 + 0.92i, ...
        8.86 +  1.81i,  -3.52 +  0.58i,   5.32 - 1.59i,   ...
       -2.83 -  0.03i,  -1.54 -  2.86i,  ...
       -0.56 + 0.12i];
b  = [-55.64 + 41.22i, -19.09 - 35.97i;
      -48.18 + 66.00i, -12.08 - 27.02i;
       -0.49 -  1.47i,   6.95 + 20.49i;
       -6.43 + 19.24i,  -4.59 - 35.53i];

% Factorize
[apf, ipiv, info] = f07qr( ...
                           uplo, n, ap);
% Solve
[x, info] = f07qs(uplo, apf, ipiv, b);

disp('Solution(s)');
disp(x);
```

9.2 Program Results

```
     f07qs example results

Solution(s)
   1.0000 - 1.0000i  -2.0000 - 1.0000i
  -2.0000 + 5.0000i   1.0000 - 3.0000i
   3.0000 - 2.0000i   3.0000 + 2.0000i
  -4.0000 + 3.0000i  -1.0000 + 1.0000i
```