

NAG Toolbox

nag_lapack_zhpcon (f07pu)

1 Purpose

nag_lapack_zhpcon (f07pu) estimates the condition number of a complex Hermitian indefinite matrix A , where A has been factorized by nag_lapack_zhptrf (f07pr), using packed storage.

2 Syntax

```
[rcond, info] = nag_lapack_zhpcon(uplo, ap, ipiv, anorm, 'n', n)
[rcond, info] = f07pu(uplo, ap, ipiv, anorm, 'n', n)
```

3 Description

nag_lapack_zhpcon (f07pu) estimates the condition number (in the 1-norm) of a complex Hermitian indefinite matrix A :

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1.$$

Since A is Hermitian, $\kappa_1(A) = \kappa_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty$.

Because $\kappa_1(A)$ is infinite if A is singular, the function actually returns an estimate of the **reciprocal** of $\kappa_1(A)$.

The function should be preceded by a computation of $\|A\|_1$ and a call to nag_lapack_zhptrf (f07pr) to compute the Bunch–Kaufman factorization of A . The function then uses Higham's implementation of Hager's method (see Higham (1988)) to estimate $\|A^{-1}\|_1$.

4 References

Higham N J (1988) FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – CHARACTER(1)

Specifies how A has been factorized.

uplo = 'U'

$A = PUDU^H P^T$, where U is upper triangular.

uplo = 'L'

$A = PLDL^H P^T$, where L is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **ap**(:) – COMPLEX (KIND=nag_wp) array

The dimension of the array **ap** must be at least $\max(1, \mathbf{n} \times (\mathbf{n} + 1)/2)$

The factorization of A stored in packed form, as returned by nag_lapack_zhptrf (f07pr).

3: **ipiv**(:) – INTEGER array

The dimension of the array **ipiv** must be at least $\max(1, \mathbf{n})$

Details of the interchanges and the block structure of D , as returned by `nag_lapack_zhptrf` (f07pr).

4: **anorm** – REAL (KIND=nag_wp)

The 1-norm of the **original** matrix A . **anorm** must be computed either **before** calling `nag_lapack_zhptrf` (f07pr) or else from a **copy** of the original matrix A .

Constraint: **anorm** ≥ 0.0 .

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the dimension of the array **ipiv**.

n , the order of the matrix A .

Constraint: **n** ≥ 0 .

5.3 Output Parameters

1: **rcond** – REAL (KIND=nag_wp)

An estimate of the reciprocal of the condition number of A . **rcond** is set to zero if exact singularity is detected or the estimate underflows. If **rcond** is less than *machine precision*, A is singular to working precision.

2: **info** – INTEGER

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info < 0

If **info** = $-i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

The computed estimate **rcond** is never less than the true value ρ , and in practice is nearly always less than 10ρ , although examples can be constructed where **rcond** is much larger.

8 Further Comments

A call to `nag_lapack_zhpcn` (f07pu) involves solving a number of systems of linear equations of the form $Ax = b$; the number is usually 5 and never more than 11. Each solution involves approximately $8n^2$ real floating-point operations but takes considerably longer than a call to `nag_lapack_zhptrs` (f07ps) with one right-hand side, because extra care is taken to avoid overflow when A is approximately singular.

The real analogue of this function is `nag_lapack_dspcn` (f07pg).

9 Example

This example estimates the condition number in the 1-norm (or ∞ -norm) of the matrix A , where

$$A = \begin{pmatrix} -1.36 + 0.00i & 1.58 + 0.90i & 2.21 - 0.21i & 3.91 + 1.50i \\ 1.58 - 0.90i & -8.87 + 0.00i & -1.84 - 0.03i & -1.78 + 1.18i \\ 2.21 + 0.21i & -1.84 + 0.03i & -4.63 + 0.00i & 0.11 + 0.11i \\ 3.91 - 1.50i & -1.78 - 1.18i & 0.11 - 0.11i & -1.84 + 0.00i \end{pmatrix}.$$

Here A is Hermitian indefinite, stored in packed form, and must first be factorized by `nag_lapack_zhprf` (f07pr). The true condition number in the 1-norm is 9.10.

9.1 Program Text

```
function f07pu_example

fprintf('f07pu example results\n\n');

% Hermitian indefinite matrix A (Lower triangular part stored in packed form)

% A in full format first to get 1-norm of A
n = nag_int(4);
a = [ -1.36 + 0.00i,   1.58 + 0.90i,   2.21 - 0.21i,   3.91 - 1.50i
      1.58 - 0.90i,  -8.87 + 0.00i,  -1.84 - 0.03i,  -1.78 + 1.18i;
      2.21 + 0.21i,  -1.84 + 0.03i,  -4.63 + 0.00i,   0.11 + 0.11i;
      3.91 - 1.50i,  -1.78 - 1.18i,   0.11 - 0.11i,  -1.84 + 0.00i];
anorm = norm(a,1);

% pack A in array ap
uplo = 'L';
ap = [];
for j = 1:n
    ap = [ap; a(j:n,j)];
end

% Factorize packed form
[apf, ipiv, info] = f07pr( ...
    uplo, n, ap);

% Condition number estimator
[rcond, info] = f07pu( ...
    uplo, apf, ipiv, anorm);

fprintf('Estimate of condition number = %9.2e\n', 1/rcond);
```

9.2 Program Results

```
f07pu example results

Estimate of condition number = 6.68e+00
```
