

NAG Toolbox

nag_lapack_dspsv (f07pa)

1 Purpose

nag_lapack_dspsv (f07pa) computes the solution to a real system of linear equations

$$AX = B,$$

where A is an n by n symmetric matrix stored in packed format and X and B are n by r matrices.

2 Syntax

```
[ap, ipiv, b, info] = nag_lapack_dspsv(uplo, ap, b, 'n', n, 'nrhs_p', nrhs_p)
[ap, ipiv, b, info] = f07pa(uplo, ap, b, 'n', n, 'nrhs_p', nrhs_p)
```

3 Description

nag_lapack_dspsv (f07pa) uses the diagonal pivoting method to factor A as $A = UDU^T$ if **uplo** = 'U' or $A = LDL^T$ if **uplo** = 'L', where U (or L) is a product of permutation and unit upper (lower) triangular matrices, D is symmetric and block diagonal with 1 by 1 and 2 by 2 diagonal blocks. The factored form of A is then used to solve the system of equations $AX = B$.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia <http://www.netlib.org/lapack/lug>

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Higham N J (2002) *Accuracy and Stability of Numerical Algorithms* (2nd Edition) SIAM, Philadelphia

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – CHARACTER(1)

If **uplo** = 'U', the upper triangle of A is stored.

If **uplo** = 'L', the lower triangle of A is stored.

Constraint: **uplo** = 'U' or 'L'.

2: **ap**(:) – REAL (KIND=nag_wp) array

The dimension of the array **ap** must be at least $\max(1, n \times (n + 1)/2)$

The n by n symmetric matrix A , packed by columns.

More precisely,

if **uplo** = 'U', the upper triangle of A must be stored with element A_{ij} in **ap**($i + j(j - 1)/2$) for $i \leq j$;

if **uplo** = 'L', the lower triangle of A must be stored with element A_{ij} in **ap**($i + (2n - j)(j - 1)/2$) for $i \geq j$.

3: **b**(*ldb*,:) – REAL (KIND=nag_wp) array

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$.

The second dimension of the array **b** must be at least $\max(1, \mathbf{nrhs_p})$.

The n by r right-hand side matrix B .

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the array **b**.

n , the number of linear equations, i.e., the order of the matrix A .

Constraint: $\mathbf{n} \geq 0$.

2: **nrhs_p** – INTEGER

Default: the second dimension of the array **b**.

r , the number of right-hand sides, i.e., the number of columns of the matrix B .

Constraint: $\mathbf{nrhs_p} \geq 0$.

5.3 Output Parameters

1: **ap**(:) – REAL (KIND=nag_wp) array

The dimension of the array **ap** will be $\max(1, \mathbf{n} \times (\mathbf{n} + 1)/2)$

The block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization $A = UDU^T$ or $A = LDL^T$ as computed by nag_lapack_dsptrf (f07pd), stored as a packed triangular matrix in the same storage format as A .

2: **ipiv**(**n**) – INTEGER array

Details of the interchanges and the block structure of D . More precisely,

if **ipiv**(i) = $k > 0$, d_{ii} is a 1 by 1 pivot block and the i th row and column of A were interchanged with the k th row and column;

if **uplo** = 'U' and **ipiv**($i - 1$) = **ipiv**(i) = $-l < 0$, $\begin{pmatrix} d_{i-1,i-1} & \bar{d}_{i,i-1} \\ \bar{d}_{i,i-1} & d_{ii} \end{pmatrix}$ is a 2 by 2 pivot block and the ($i - 1$)th row and column of A were interchanged with the l th row and column;

if **uplo** = 'L' and **ipiv**(i) = **ipiv**($i + 1$) = $-m < 0$, $\begin{pmatrix} d_{ii} & d_{i+1,i} \\ d_{i+1,i} & d_{i+1,i+1} \end{pmatrix}$ is a 2 by 2 pivot block and the ($i + 1$)th row and column of A were interchanged with the m th row and column.

3: **b**(*ldb*,:) – REAL (KIND=nag_wp) array

The first dimension of the array **b** will be $\max(1, \mathbf{n})$.

The second dimension of the array **b** will be $\max(1, \mathbf{nrhs_p})$.

If **info** = 0, the n by r solution matrix X .

4: **info** – INTEGER

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info < 0

If **info** = $-i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

info > 0 (*warning*)

Element $\langle value \rangle$ of the diagonal is exactly zero. The factorization has been completed, but the block diagonal matrix D is exactly singular, so the solution could not be computed.

7 Accuracy

The computed solution for a single right-hand side, \hat{x} , satisfies an equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_1 = O(\epsilon)\|A\|_1$$

and ϵ is the *machine precision*. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x} - x\|_1}{\|x\|_1} \leq \kappa(A) \frac{\|E\|_1}{\|A\|_1},$$

where $\kappa(A) = \|A^{-1}\|_1 \|A\|_1$, the condition number of A with respect to the solution of the linear equations. See Section 4.4 of Anderson *et al.* (1999) and Chapter 11 of Higham (2002) for further details.

nag_lapack_dspsvx (f07pb) is a comprehensive LAPACK driver that returns forward and backward error bounds and an estimate of the condition number. Alternatively, nag_linsys_real_symm_packed_solve (f04bj) solves $AX = B$ and returns a forward error bound and condition estimate. nag_linsys_real_symm_packed_solve (f04bj) calls nag_lapack_dspsv (f07pa) to solve the equations.

8 Further Comments

The total number of floating-point operations is approximately $\frac{1}{3}n^3 + 2n^2r$, where r is the number of right-hand sides.

The complex analogues of nag_lapack_dspsv (f07pa) are nag_lapack_zhpsv (f07pn) for Hermitian matrices, and nag_lapack_zspsv (f07qn) for symmetric matrices.

9 Example

This example solves the equations

$$Ax = b,$$

where A is the symmetric matrix

$$A = \begin{pmatrix} -1.81 & 2.06 & 0.63 & -1.15 \\ 2.06 & 1.15 & 1.87 & 4.20 \\ 0.63 & 1.87 & -0.21 & 3.87 \\ -1.15 & 4.20 & 3.87 & 2.07 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 0.96 \\ 6.07 \\ 8.38 \\ 9.50 \end{pmatrix}.$$

Details of the factorization of A are also output.

9.1 Program Text

```
function f07pa_example

fprintf('f07pa example results\n\n');

% Indefinite matrix A, Upper triangle stored in packed form
uplo = 'Upper';
n = nag_int(4);
ap = [-1.81 ...
      2.06 1.15 ...
      0.63 1.87 -0.21 ...
      -1.15 4.20 3.87 2.07];

% RHS
b = [0.96;
     6.07;
     8.38;
     9.50];

[apf, ipiv, x, info] = f07pa( ...
                      uplo, ap, b);

disp('Solution');
disp(x');

[ifail] = x04cc( ...
              uplo, 'Non-unit', n, apf, 'Details of factorization');

fprintf('\nPivot indices\n ');
fprintf('%11d', ipiv);
fprintf('\n');
```

9.2 Program Results

```
f07pa example results

Solution
-5.0000  -2.0000  1.0000  4.0000

Details of factorization
          1          2          3          4
1      0.4074      0.3031     -0.5960      0.6537
2          -2.5907      0.8115      0.2230
3          1.1500      4.2000
4          2.0700

Pivot indices
          1          2          -2          -2
```
