NAG Toolbox

nag_lapack_dptcon (f07jg)

1 Purpose

nag_lapack_dptcon (f07jg) computes the reciprocal condition number of a real n by n symmetric positive definite tridiagonal matrix A, using the LDL^{T} factorization returned by nag_lapack_dpttrf (f07jd).

2 Syntax

```
[rcond, info] = nag_lapack_dptcon(d, e, anorm, 'n', n)
[rcond, info] = f07jg(d, e, anorm, 'n', n)
```

3 Description

nag_lapack_dptcon (f07jg) should be preceded by a call to nag_lapack_dpttrf (f07jd), which computes a modified Cholesky factorization of the matrix A as

$$A = LDL^{\mathrm{T}},$$

where L is a unit lower bidiagonal matrix and D is a diagonal matrix, with positive diagonal elements. nag_lapack_dptcon (f07jg) then utilizes the factorization to compute $\left\|A^{-1}\right\|_1$ by a direct method, from which the reciprocal of the condition number of A, $1/\kappa(A)$ is computed as

$$1/\kappa_1(A) = 1/\left(\|A\|_1\left\|A^{-1}\right\|_1\right).$$

$1/\kappa(A)$ is returned, rather than $\kappa(A)$, since when A is singular $\kappa(A)$ is infinite.

4 References

Higham N J (2002) *Accuracy and Stability of Numerical Algorithms* (2nd Edition) SIAM, Philadelphia

5 Parameters

5.1 Compulsory Input Parameters

1: **d**(:) – REAL (KIND=nag_wp) array

The dimension of the array **d** must be at least $\max(1, \mathbf{n})$

Must contain the n diagonal elements of the diagonal matrix D from the LDL^{T} factorization of A.

2: **e**(:) – REAL (KIND=nag_wp) array

The dimension of the array **e** must be at least $\max(1, \mathbf{n} - 1)$

Must contain the $(n-1)$ subdiagonal elements of the unit lower bidiagonal matrix L. (**e** can also be regarded as the superdiagonal of the unit upper bidiagonal matrix U from the $U^{\mathrm{T}}DU$ factorization of A.)

3: **anorm** – REAL (KIND=nag_wp)

The 1-norm of the **original** matrix A. **anorm** must be computed either **before** calling nag_lapack_dpttrf (f07jd) or else from a **copy** of the original matrix A.

Constraint: **anorm** ≥ 0.0.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the dimension of the array **d**.

n, the order of the matrix A.

Constraint: **n** ≥ 0.

5.3 Output Parameters

1: **rcond** – REAL (KIND=nag_wp)

The reciprocal condition number, $1/\kappa_1(A) = 1/\left(\|A\|_1 \|A^{-1}\|_1\right)$.

2: **info** – INTEGER

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info < 0

If **info** $= -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

The computed condition number will be the exact condition number for a closely neighbouring matrix.

8 Further Comments

The condition number estimation requires $O(n)$ floating-point operations.

See Section 15.6 of Higham (2002) for further details on computing the condition number of tridiagonal matrices.

The complex analogue of this function is nag_lapack_zptcon (f07ju).

9 Example

This example computes the condition number of the symmetric positive definite tridiagonal matrix A given by

$$A = \begin{pmatrix} 4.0 & -2.0 & 0 & 0 & 0 \\ -2.0 & 10.0 & -6.0 & 0 & 0 \\ 0 & -6.0 & 29.0 & 15.0 & 0 \\ 0 & 0 & 15.0 & 25.0 & 8.0 \\ 0 & 0 & 0 & 8.0 & 5.0 \end{pmatrix}.$$

9.1 Program Text

```
     function f07jg_example

fprintf('f07jg example results\n\n');

% Symmetric tridiagonal A stored as two diagonals
d = [ 4     10     29     25     5];
e = [-2     -6     15     8       ];

[df, ef, info] = f07jd( ...
                        d, e);

% Construct matrix an with same 1-norm
an = [0 e; d; e 0];
anorm = norm(an,1);

% Get reciprocal condition number
[rcond, info] = f07jg( ...
                       df, ef, anorm);

fprintf('Condition number of A = %7.2e\n',1/rcond);
```

9.2 Program Results

```
     f07jg example results

Condition number of A = 1.05e+02
```