NAG Toolbox

nag_lapack_zpbcon (f07hu)

1 Purpose

nag_lapack_zpbcon (f07hu) estimates the condition number of a complex Hermitian positive definite band matrix A, where A has been factorized by nag_lapack_zpbtrf (f07hr).

2 Syntax

```
[rcond, info] = nag_lapack_zpbcon(uplo, kd, ab, anorm, 'n', n)
[rcond, info] = f07hu(uplo, kd, ab, anorm, 'n', n)
```

3 Description

nag_lapack_zpbcon (f07hu) estimates the condition number (in the 1-norm) of a complex Hermitian positive definite band matrix A:

$$\kappa_1(A) = \|A\|_1 \left\|A^{-1}\right\|_1.$$

Since A is Hermitian, $\kappa_1(A) = \kappa_\infty(A) = \|A\|_\infty \left\|A^{-1}\right\|_\infty$.

Because $\kappa_1(A)$ is infinite if A is singular, the function actually returns an estimate of the **reciprocal** of $\kappa_1(A)$.

The function should be preceded by a computation of $\|A\|_1$ and a call to nag_lapack_zpbtrf (f07hr) to compute the Cholesky factorization of A. The function then uses Higham's implementation of Hager's method (see Higham (1988)) to estimate $\left\|A^{-1}\right\|_1$.

4 References

Higham N J (1988) FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – CHARACTER(1)

Specifies how A has been factorized.

uplo = 'U'
$A = U^{\mathrm{H}}U$, where U is upper triangular.

uplo = 'L'
$A = LL^{\mathrm{H}}$, where L is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **kd** – INTEGER

k_d, the number of superdiagonals or subdiagonals of the matrix A.

Constraint: **kd** ≥ 0.

3: **ab**$(ldab,:)$ – COMPLEX (KIND=nag_wp) array

The first dimension of the array **ab** must be at least $\mathbf{kd}+1$.

The second dimension of the array **ab** must be at least $\max(1,\mathbf{n})$.

The Cholesky factor of A, as returned by nag_lapack_zpbtrf (f07hr).

4: **anorm** – REAL (KIND=nag_wp)

The 1-norm of the **original** matrix A. **anorm** must be computed either **before** calling nag_lapack_zpbtrf (f07hr) or else from a **copy** of the original matrix A.

Constraint: $\mathbf{anorm} \geq 0.0$.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the second dimension of the array **ab**.

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5.3 Output Parameters

1: **rcond** – REAL (KIND=nag_wp)

An estimate of the reciprocal of the condition number of A. **rcond** is set to zero if exact singularity is detected or the estimate underflows. If **rcond** is less than ***machine precision***, A is singular to working precision.

2: **info** – INTEGER

$\mathbf{info} = 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

$\mathbf{info} < 0$

If $\mathbf{info} = -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

The computed estimate **rcond** is never less than the true value ρ, and in practice is nearly always less than 10ρ, although examples can be constructed where **rcond** is much larger.

8 Further Comments

A call to nag_lapack_zpbcon (f07hu) involves solving a number of systems of linear equations of the form $Ax = b$; the number is usually 5 and never more than 11. Each solution involves approximately $16nk$ real floating-point operations (assuming $n \gg k$) but takes considerably longer than a call to nag_lapack_zpbtrs (f07hs) with one right-hand side, because extra care is taken to avoid overflow when A is approximately singular.

The real analogue of this function is nag_lapack_dpbcon (f07hg).

9 Example

This example estimates the condition number in the 1-norm (or ∞-norm) of the matrix A, where

$$A = \begin{pmatrix} 9.39 + 0.00i & 1.08 - 1.73i & 0.00 + 0.00i & 0.00 + 0.00i \\ 1.08 + 1.73i & 1.69 + 0.00i & -0.04 + 0.29i & 0.00 + 0.00i \\ 0.00 + 0.00i & -0.04 - 0.29i & 2.65 + 0.00i & -0.33 + 2.24i \\ 0.00 + 0.00i & 0.00 + 0.00i & -0.33 - 2.24i & 2.17 + 0.00i \end{pmatrix}.$$

Here A is Hermitian positive definite, and is treated as a band matrix, which must first be factorized by nag_lapack_zpbtrf (f07hr). The true condition number in the 1-norm is 153.45.

9.1 Program Text

```
     function f07hu_example

fprintf('f07hu example results\n\n');

uplo = 'L';
kd = nag_int(1);
n  = nag_int(4);
ab = [ 9.39 + 0i,     1.69 + 0i,      2.65 + 0i,      2.17 + 0i;
       1.08 - 1.73i, -0.04 + 0.29i,  -0.33 + 2.24i    0    + 0i];

% Factorize
[abf, info] = f07hr( ...
                     uplo, kd, ab);

% To calculate 1-norm here, need to add superdiagonal
abn = [0 + 0i,  ab(2,1:n-1);
       ab];
% 1-norm of A = 1-norm of abn
  anorm = norm(abn,1);

% Get reciprocal condition number
[rcond, info] = f07hu( ...
                       uplo, kd, abf, anorm);

fprintf('Estimate of Condition number = %7.2e\n',1/rcond);
```

9.2 Program Results

```
     f07hu example results

Estimate of Condition number = 1.32e+02
```