NAG Toolbox

nag_lapack_zpbtrf (f07hr)

1 Purpose

nag_lapack_zpbtrf (f07hr) computes the Cholesky factorization of a complex Hermitian positive definite band matrix.

2 Syntax

```
[ab, info] = nag_lapack_zpbtrf(uplo, kd, ab, 'n', n)
[ab, info] = f07hr(uplo, kd, ab, 'n', n)
```

3 Description

nag_lapack_zpbtrf (f07hr) forms the Cholesky factorization of a complex Hermitian positive definite band matrix A either as $A = U^{H}U$ if **uplo** $=$ 'U' or $A = LL^{H}$ if **uplo** $=$ 'L', where U (or L) is an upper (or lower) triangular band matrix with the same number of superdiagonals (or subdiagonals) as A.

4 References

Demmel J W (1989) On floating-point errors in Cholesky *LAPACK Working Note No. 14* University of Tennessee, Knoxville http://www.netlib.org/lapack/lawnspdf/lawn14.pdf

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – CHARACTER(1)

Specifies whether the upper or lower triangular part of A is stored and how A is to be factorized.

uplo $=$ 'U'
The upper triangular part of A is stored and A is factorized as $U^{H}U$, where U is upper triangular.

uplo $=$ 'L'
The lower triangular part of A is stored and A is factorized as LL^{H}, where L is lower triangular.

Constraint: **uplo** $=$ 'U' or 'L'.

2: **kd** – INTEGER

k_d, the number of superdiagonals or subdiagonals of the matrix A.

Constraint: **kd** ≥ 0.

3: **ab**$(ldab,:)$ – COMPLEX (KIND=nag_wp) array

The first dimension of the array **ab** must be at least **kd** $+ 1$.

The second dimension of the array **ab** must be at least $\max(1, \mathbf{n})$.

The n by n Hermitian positive definite band matrix A.

The matrix is stored in rows 1 to $k_d + 1$, more precisely,

if **uplo** = 'U', the elements of the upper triangle of A within the band must be stored with element A_{ij} in **ab**$(k_d + 1 + i - j, j)$ for $\max(1, j - k_d) \leq i \leq j$;

if **uplo** = 'L', the elements of the lower triangle of A within the band must be stored with element A_{ij} in **ab**$(1 + i - j, j)$ for $j \leq i \leq \min(n, j + k_d)$.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the second dimension of the array **ab**.

n, the order of the matrix A.

Constraint: **n** ≥ 0.

5.3 Output Parameters

1: **ab**$(ldab, :)$ – COMPLEX (KIND=nag_wp) array

The first dimension of the array **ab** will be **kd** $+ 1$.

The second dimension of the array **ab** will be $\max(1, \mathbf{n})$.

The upper or lower triangle of A stores the Cholesky factor U or L as specified by **uplo**, using the same storage format as described above.

2: **info** – INTEGER

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info < 0

If **info** $= -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

info > 0

The leading minor of order $\langle value \rangle$ is not positive definite and the factorization could not be completed. Hence A itself is not positive definite. This may indicate an error in forming the matrix A. There is no function specifically designed to factorize a Hermitian band matrix which is not positive definite; the matrix must be treated either as a nonsymmetric band matrix, by calling nag_lapack_zgbtrf (f07br) or as a full Hermitian matrix, by calling nag_lapack_zhetrf (f07mr).

7 Accuracy

If **uplo** = 'U', the computed factor U is the exact factor of a perturbed matrix $A + E$, where

$$|E| \leq c(k+1)\epsilon |U^{\mathrm{H}}||U|,$$

$c(k+1)$ is a modest linear function of $k+1$, and ϵ is the ***machine precision***.

If **uplo** = 'L', a similar statement holds for the computed factor L. It follows that $|e_{ij}| \leq c(k+1)\epsilon\sqrt{a_{ii}a_{jj}}$.

8 Further Comments

The total number of real floating-point operations is approximately $4n(k+1)^2$, assuming $n \gg k$.

A call to nag_lapack_zpbtrf (f07hr) may be followed by calls to the functions:

nag_lapack_zpbtrs (f07hs) to solve $AX = B$;

nag_lapack_zpbcon (f07hu) to estimate the condition number of A.

The real analogue of this function is nag_lapack_dpbtrf (f07hd).

9 Example

This example computes the Cholesky factorization of the matrix A, where

$$A = \begin{pmatrix} 9.39 + 0.00i & 1.08 - 1.73i & 0.00 + 0.00i & 0.00 + 0.00i \\ 1.08 + 1.73i & 1.69 + 0.00i & -0.04 + 0.29i & 0.00 + 0.00i \\ 0.00 + 0.00i & -0.04 - 0.29i & 2.65 + 0.00i & -0.33 + 2.24i \\ 0.00 + 0.00i & 0.00 + 0.00i & -0.33 - 2.24i & 2.17 + 0.00i \end{pmatrix}.$$

9.1 Program Text

```
function f07hr_example

fprintf('f07hr example results\n\n');

uplo = 'L';
kd = nag_int(1);
n  = nag_int(4);
ab = [ 9.39 + 0i,     1.69 + 0i,      2.65 + 0i,      2.17 + 0i;
       1.08 - 1.73i, -0.04 + 0.29i,  -0.33 + 2.24i    0    + 0i];

% Factorize
[abf, info] = f07hr( ...
                     uplo, kd, ab);

ku = nag_int(0);
[ifail] = x04de( ...
                 n, n, kd, ku, abf, 'Cholesky factor');
```

9.2 Program Results

```
f07hr example results

 Cholesky factor
            1          2          3          4
 1      3.0643
        0.0000

 2      0.3524     1.1167
       -0.5646     0.0000

 3                -0.0358     1.6066
                   0.2597     0.0000

 4                           -0.2054     0.4289
                              1.3942     0.0000
```