NAG Toolbox

nag_lapack_dpptri (f07gj)

1 Purpose

nag_lapack_dpptri (f07gj) computes the inverse of a real symmetric positive definite matrix A, where A has been factorized by nag_lapack_dpptrf (f07gd), using packed storage.

2 Syntax

```
[ap, info] = nag_lapack_dpptri(uplo, n, ap)
[ap, info] = f07gj(uplo, n, ap)
```

3 Description

nag_lapack_dpptri (f07gj) is used to compute the inverse of a real symmetric positive definite matrix A, the function must be preceded by a call to nag_lapack_dpptrf (f07gd), which computes the Cholesky factorization of A, using packed storage.

If **uplo** = 'U', $A = U^{\mathrm{T}}U$ and A^{-1} is computed by first inverting U and then forming $(U^{-1})U^{-\mathrm{T}}$.

If **uplo** = 'L', $A = LL^{\mathrm{T}}$ and A^{-1} is computed by first inverting L and then forming $L^{-\mathrm{T}}(L^{-1})$.

4 References

Du Croz J J and Higham N J (1992) Stability of methods for matrix inversion *IMA J. Numer. Anal.* **12** 1–19

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – CHARACTER(1)

Specifies how A has been factorized.

uplo = 'U'
$A = U^{\mathrm{T}}U$, where U is upper triangular.

uplo = 'L'
$A = LL^{\mathrm{T}}$, where L is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **n** – INTEGER

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

3: **ap**(:) – REAL (KIND=nag_wp) array

The dimension of the array **ap** must be at least $\max(1, \mathbf{n} \times (\mathbf{n}+1)/2)$

The Cholesky factor of A stored in packed form, as returned by nag_lapack_dpptrf (f07gd).

5.2 Optional Input Parameters

None.

5.3 Output Parameters

1: **ap**(:) – REAL (KIND=nag_wp) array

The dimension of the array **ap** will be $\max(1, \mathbf{n} \times (\mathbf{n}+1)/2)$

The factorization stores the n by n matrix A^{-1}.

More precisely,

if **uplo** = 'U', the upper triangle of A^{-1} must be stored with element A_{ij} in **ap**$(i + j(j-1)/2)$ for $i \leq j$;

if **uplo** = 'L', the lower triangle of A^{-1} must be stored with element A_{ij} in **ap**$(i + (2n-j)(j-1)/2)$ for $i \geq j$.

2: **info** – INTEGER

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info < 0

If **info** $= -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

info > 0 (*warning*)

Diagonal element $\langle value \rangle$ of the Cholesky factor is zero; the Cholesky factor is singular and the inverse of A cannot be computed.

7 Accuracy

The computed inverse X satisfies

$$\|XA - I\|_2 \leq c(n)\epsilon\kappa_2(A) \quad \text{and} \quad \|AX - I\|_2 \leq c(n)\epsilon\kappa_2(A),$$

where $c(n)$ is a modest function of n, ϵ is the ***machine precision*** and $\kappa_2(A)$ is the condition number of A defined by

$$\kappa_2(A) = \|A\|_2 \left\|A^{-1}\right\|_2.$$

8 Further Comments

The total number of floating-point operations is approximately $\frac{2}{3}n^3$.

The complex analogue of this function is nag_lapack_zpptri (f07gw).

9 Example

This example computes the inverse of the matrix A, where

$$A = \begin{pmatrix} 4.16 & -3.12 & 0.56 & -0.10 \\ -3.12 & 5.03 & -0.83 & 1.18 \\ 0.56 & -0.83 & 0.76 & 0.34 \\ -0.10 & 1.18 & 0.34 & 1.18 \end{pmatrix}.$$

Here A is symmetric positive definite, stored in packed form, and must first be factorized by nag_lapack_dpptrf (f07gd).

9.1 Program Text

```
     function f07gj_example

fprintf('f07gj example results\n\n');

% Symmetric matrix A, lower triangular part packed in ap
uplo = 'L';
n = nag_int(4);
ap = [4.16 -3.12  0.56 -0.10 ...
           5.03 -0.83  1.18 ...
                0.76  0.34 ...
                      1.18];

[L, info] = f07gd( ...
                  uplo, n, ap);

[ainv, info] = f07gj( ...
                     uplo, n, L);

[ifail] = x04cc( ...
                uplo, 'N', n, ainv, 'Inverse');
```

9.2 Program Results

```
     f07gj example results

 Inverse
             1          2          3          4
 1      0.6995
 2      0.7769     1.4239
 3      0.7508     1.8255     4.0688
 4     -0.9340    -1.8841    -2.9342     3.4978
```