NAG Toolbox

nag_lapack_dpptrf (f07gd)

1 Purpose

nag_lapack_dpptrf (f07gd) computes the Cholesky factorization of a real symmetric positive definite matrix, using packed storage.

2 Syntax

```
[ap, info] = nag_lapack_dpptrf(uplo, n, ap)
[ap, info] = f07gd(uplo, n, ap)
```

3 Description

nag_lapack_dpptrf (f07gd) forms the Cholesky factorization of a real symmetric positive definite matrix A either as $A = U^{\mathrm{T}}U$ if **uplo** = 'U' or $A = LL^{\mathrm{T}}$ if **uplo** = 'L', where U is an upper triangular matrix and L is lower triangular, using packed storage.

4 References

Demmel J W (1989) On floating-point errors in Cholesky *LAPACK Working Note No. 14* University of Tennessee, Knoxville http://www.netlib.org/lapack/lawnspdf/lawn14.pdf

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – CHARACTER(1)

Specifies whether the upper or lower triangular part of A is stored and how A is to be factorized.

uplo = 'U'
The upper triangular part of A is stored and A is factorized as $U^{\mathrm{T}}U$, where U is upper triangular.

uplo = 'L'
The lower triangular part of A is stored and A is factorized as LL^{T}, where L is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **n** – INTEGER

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

3: **ap**(:) – REAL (KIND=nag_wp) array

The dimension of the array **ap** must be at least $\max(1, \mathbf{n} \times (\mathbf{n}+1)/2)$

The n by n symmetric matrix A, packed by columns.

More precisely,

if **uplo** = 'U', the upper triangle of A must be stored with element A_{ij} in **ap**$(i + j(j-1)/2)$ for $i \leq j$;

if **uplo** = 'L', the lower triangle of A must be stored with element A_{ij} in **ap**$(i + (2n-j)(j-1)/2)$ for $i \geq j$.

5.2 Optional Input Parameters

None.

5.3 Output Parameters

1: **ap**(:) – REAL (KIND=nag_wp) array

The dimension of the array **ap** will be $\max(1, \mathbf{n} \times (\mathbf{n}+1)/2)$

If **info** $= 0$, the factor U or L from the Cholesky factorization $A = U^{\mathrm{T}}U$ or $A = LL^{\mathrm{T}}$, in the same storage format as A.

2: **info** – INTEGER

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info < 0

If **info** $= -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

info > 0

The leading minor of order ⟨*value*⟩ is not positive definite and the factorization could not be completed. Hence A itself is not positive definite. This may indicate an error in forming the matrix A. To factorize a symmetric matrix which is not positive definite, call nag_lapack_dsptrf (f07pd) instead.

7 Accuracy

If **uplo** = 'U', the computed factor U is the exact factor of a perturbed matrix $A + E$, where

$$|E| \leq c(n)\epsilon \left|U^{\mathrm{T}}\right||U|,$$

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

If **uplo** = 'L', a similar statement holds for the computed factor L. It follows that $\left|e_{ij}\right| \leq c(n)\epsilon\sqrt{a_{ii}a_{jj}}$.

8 Further Comments

The total number of floating-point operations is approximately $\frac{1}{3}n^3$.

A call to nag_lapack_dpptrf (f07gd) may be followed by calls to the functions:

nag_lapack_dpptrs (f07ge) to solve $AX = B$;

nag_lapack_dppcon (f07gg) to estimate the condition number of A;

nag_lapack_dpptri (f07gj) to compute the inverse of A.

The complex analogue of this function is nag_lapack_zpptrf (f07gr).

9 Example

This example computes the Cholesky factorization of the matrix A, where

$$A = \begin{pmatrix} 4.16 & -3.12 & 0.56 & -0.10 \\ -3.12 & 5.03 & -0.83 & 1.18 \\ 0.56 & -0.83 & 0.76 & 0.34 \\ -0.10 & 1.18 & 0.34 & 1.18 \end{pmatrix},$$

using packed storage.

9.1 Program Text

```
     function f07gd_example

fprintf('f07gd example results\n\n');

% Symmetric matrix A, lower triangular part packed in ap
uplo = 'L';
n = nag_int(4);
ap = [4.16 -3.12  0.56 -0.10 ...
           5.03 -0.83  1.18 ...
                0.76  0.34 ...
                      1.18];

[L, info] = f07gd( ...
                  uplo, n, ap);

[ifail] = x04cc( ...
                uplo, 'N', n, L, 'Cholesky factor L');
```

9.2 Program Results

```
     f07gd example results

 Cholesky factor L
            1          2          3          4
 1      2.0396
 2     -1.5297     1.6401
 3      0.2746    -0.2500     0.7887
 4     -0.0490     0.6737     0.6617     0.5347
```