NAG Toolbox

nag_lapack_zpotrf (f07fr)

1 Purpose

nag_lapack_zpotrf (f07fr) computes the Cholesky factorization of a complex Hermitian positive definite matrix.

2 Syntax

```
[a, info] = nag_lapack_zpotrf(uplo, a, 'n', n)
[a, info] = f07fr(uplo, a, 'n', n)
```

3 Description

nag_lapack_zpotrf (f07fr) forms the Cholesky factorization of a complex Hermitian positive definite matrix A either as $A = U^{\mathrm{H}}U$ if **uplo** = 'U' or $A = LL^{\mathrm{H}}$ if **uplo** = 'L', where U is an upper triangular matrix and L is lower triangular.

4 References

Demmel J W (1989) On floating-point errors in Cholesky *LAPACK Working Note No. 14* University of Tennessee, Knoxville http://www.netlib.org/lapack/lawnspdf/lawn14.pdf

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – CHARACTER(1)

Specifies whether the upper or lower triangular part of A is stored and how A is to be factorized.

uplo = 'U'
The upper triangular part of A is stored and A is factorized as $U^{\mathrm{H}}U$, where U is upper triangular.

uplo = 'L'
The lower triangular part of A is stored and A is factorized as LL^{H}, where L is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **a**$(lda,:)$ – COMPLEX (KIND=nag_wp) array

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$.

The second dimension of the array **a** must be at least $\max(1, \mathbf{n})$.

The n by n Hermitian positive definite matrix A.

If **uplo** = 'U', the upper triangular part of a must be stored and the elements of the array below the diagonal are not referenced.

If **uplo** = 'L', the lower triangular part of a must be stored and the elements of the array above the diagonal are not referenced.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the array **a** and the second dimension of the array **a**.

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5.3 Output Parameters

1: **a**$(lda,:)$ – COMPLEX (KIND=nag_wp) array

The first dimension of the array **a** will be $\max(1,\mathbf{n})$.

The second dimension of the array **a** will be $\max(1,\mathbf{n})$.

The upper or lower triangle of A stores the Cholesky factor U or L as specified by **uplo**.

2: **info** – INTEGER

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info < 0

If **info** $= -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

info > 0

The leading minor of order $\langle value \rangle$ is not positive definite and the factorization could not be completed. Hence A itself is not positive definite. This may indicate an error in forming the matrix A. To factorize a Hermitian matrix which is not positive definite, call nag_lapack_zhetrf (f07mr) instead.

7 Accuracy

If **uplo** = 'U', the computed factor U is the exact factor of a perturbed matrix $A + E$, where

$$|E| \leq c(n)\epsilon \left|U^{\mathrm{H}}\right||U|,$$

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***. If **uplo** = 'L', a similar statement holds for the computed factor L. It follows that $\left|e_{ij}\right| \leq c(n)\epsilon\sqrt{a_{ii}a_{jj}}$.

8 Further Comments

The total number of real floating-point operations is approximately $\frac{4}{3}n^3$.

A call to nag_lapack_zpotrf (f07fr) may be followed by calls to the functions:

nag_lapack_zpotrs (f07fs) to solve $AX = B$;

nag_lapack_zpocon (f07fu) to estimate the condition number of A;

nag_lapack_zpotri (f07fw) to compute the inverse of A.

The real analogue of this function is nag_lapack_dpotrf (f07fd).

9 Example

This example computes the Cholesky factorization of the matrix A, where

$$A = \begin{pmatrix} 3.23 + 0.00i & 1.51 - 1.92i & 1.90 + 0.84i & 0.42 + 2.50i \\ 1.51 + 1.92i & 3.58 + 0.00i & -0.23 + 1.11i & -1.18 + 1.37i \\ 1.90 - 0.84i & -0.23 - 1.11i & 4.09 + 0.00i & 2.33 - 0.14i \\ 0.42 - 2.50i & -1.18 - 1.37i & 2.33 + 0.14i & 4.29 + 0.00i \end{pmatrix}.$$

9.1 Program Text

```
function f07fr_example

fprintf('f07fr example results\n\n');

% Lower triangular part of Hermitian matrix A
uplo = 'Lower';
a = [ 3.23 + 0i,     0    + 0i,     0    + 0i,     0    + 0i;
      1.51 + 1.92i,  3.58 + 0i,     0    + 0i,     0    + 0i;
      1.90 - 0.84i, -0.23 - 1.11i,  4.09 + 0i,     0    + 0i;
      0.42 - 2.50i, -1.18 - 1.37i,  2.33 + 0.14i,  4.29 + 0i];

[L, info] = f07fr( ...
                   uplo, a);

[ifail] = x04da( ...
                 uplo, 'Non-unit', L, 'factor');
```

9.2 Program Results

```
f07fr example results

 factor
             1          2          3          4
 1      1.7972
        0.0000

 2      0.8402     1.3164
        1.0683     0.0000

 3      1.0572    -0.4702     1.5604
       -0.4674     0.3131     0.0000

 4      0.2337     0.0834     0.9360     0.6603
       -1.3910     0.0368     0.9900     0.0000
```