

NAG Toolbox

nag_lapack_dpotri (f07fj)

1 Purpose

nag_lapack_dpotri (f07fj) computes the inverse of a real symmetric positive definite matrix A , where A has been factorized by nag_lapack_dpotrf (f07fd).

2 Syntax

```
[a, info] = nag_lapack_dpotri(uplo, a, 'n', n)
[a, info] = f07fj(uplo, a, 'n', n)
```

3 Description

nag_lapack_dpotri (f07fj) is used to compute the inverse of a real symmetric positive definite matrix A , the function must be preceded by a call to nag_lapack_dpotrf (f07fd), which computes the Cholesky factorization of A .

If **uplo** = 'U', $A = U^T U$ and A^{-1} is computed by first inverting U and then forming $(U^{-1})U^{-T}$.

If **uplo** = 'L', $A = LL^T$ and A^{-1} is computed by first inverting L and then forming $L^{-T}(L^{-1})$.

4 References

Du Croz J J and Higham N J (1992) Stability of methods for matrix inversion *IMA J. Numer. Anal.* **12** 1–19

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – CHARACTER(1)

Specifies how A has been factorized.

uplo = 'U'

$A = U^T U$, where U is upper triangular.

uplo = 'L'

$A = LL^T$, where L is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **a(lda, :)** – REAL (KIND=nag_wp) array

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$.

The second dimension of the array **a** must be at least $\max(1, \mathbf{n})$.

The upper triangular matrix U if **uplo** = 'U' or the lower triangular matrix L if **uplo** = 'L', as returned by nag_lapack_dpotrf (f07fd).

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the array **a** and the second dimension of the array **a**.

n , the order of the matrix A .

Constraint: $\mathbf{n} \geq 0$.

5.3 Output Parameters

1: $\mathbf{a}(\mathit{lda}, :)$ – REAL (KIND=nag_wp) array

The first dimension of the array \mathbf{a} will be $\max(1, \mathbf{n})$.

The second dimension of the array \mathbf{a} will be $\max(1, \mathbf{n})$.

U stores the upper triangle of A^{-1} if $\mathbf{uplo} = 'U'$; L stores the lower triangle of A^{-1} if $\mathbf{uplo} = 'L'$.

2: \mathbf{info} – INTEGER

$\mathbf{info} = 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

$\mathbf{info} < 0$

If $\mathbf{info} = -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

$\mathbf{info} > 0$ (*warning*)

Diagonal element $\langle \mathit{value} \rangle$ of the Cholesky factor is zero; the Cholesky factor is singular and the inverse of A cannot be computed.

7 Accuracy

The computed inverse X satisfies

$$\|XA - I\|_2 \leq c(n)\epsilon\kappa_2(A) \quad \text{and} \quad \|AX - I\|_2 \leq c(n)\epsilon\kappa_2(A),$$

where $c(n)$ is a modest function of n , ϵ is the *machine precision* and $\kappa_2(A)$ is the condition number of A defined by

$$\kappa_2(A) = \|A\|_2 \|A^{-1}\|_2.$$

8 Further Comments

The total number of floating-point operations is approximately $\frac{2}{3}n^3$.

The complex analogue of this function is nag_lapack_zpotri (f07fw).

9 Example

This example computes the inverse of the matrix A , where

$$A = \begin{pmatrix} 4.16 & -3.12 & 0.56 & -0.10 \\ -3.12 & 5.03 & -0.83 & 1.18 \\ 0.56 & -0.83 & 0.76 & 0.34 \\ -0.10 & 1.18 & 0.34 & 1.18 \end{pmatrix}.$$

Here A is symmetric positive definite and must first be factorized by nag_lapack_dpotrf (f07fd).

9.1 Program Text

```
function f07fj_example
fprintf('f07fj example results\n\n');

% Lower triangular part of symmetric matrix A
uplo = 'Lower';
a = [ 4.16,  0,    0,    0;
      -3.12, 5.03, 0,    0;
        0.56, -0.83, 0.76, 0;
       -0.10, 1.18, 0.34, 1.18];

% Factorize A
[L, info] = f07fd( ...
               uplo, a);

% Invert A using L.
[ainv, info] = f07fj( ...
                  uplo, L);

[ifail] = x04ca( ...
               uplo, 'N', ainv, 'Inverse');
```

9.2 Program Results

```
f07fj example results

Inverse
      1          2          3          4
1      0.6995
2      0.7769      1.4239
3      0.7508      1.8255      4.0688
4     -0.9340     -1.8841     -2.9342      3.4978
```
