NAG Toolbox

nag_lapack_dgbtrs (f07be)

1 Purpose

nag_lapack_dgbtrs (f07be) solves a real band system of linear equations with multiple right-hand sides,

$$AX = B \quad \text{or} \quad A^{\mathrm{T}}X = B,$$

where A has been factorized by nag_lapack_dgbtrf (f07bd).

2 Syntax

```
[b, info] = nag_lapack_dgbtrs(trans, kl, ku, ab, ipiv, b, 'n', n, 'nrhs_p',
nrhs_p)
```

```
[b, info] = f07be(trans, kl, ku, ab, ipiv, b, 'n', n, 'nrhs_p', nrhs_p)
```

3 Description

nag_lapack_dgbtrs (f07be) is used to solve a real band system of linear equations $AX = B$ or $A^{\mathrm{T}}X = B$, the function must be preceded by a call to nag_lapack_dgbtrf (f07bd) which computes the LU factorization of A as $A = PLU$. The solution is computed by forward and backward substitution.

If **trans** $=$ 'N', the solution is computed by solving $PLY = B$ and then $UX = Y$.

If **trans** $=$ 'T' or 'C', the solution is computed by solving $U^{\mathrm{T}}Y = B$ and then $L^{\mathrm{T}}P^{\mathrm{T}}X = Y$.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **trans** – CHARACTER(1)

Indicates the form of the equations.

trans $=$ 'N'
$AX = B$ is solved for X.

trans $=$ 'T' or 'C'
$A^{\mathrm{T}}X = B$ is solved for X.

Constraint: **trans** $=$ 'N', 'T' or 'C'.

2: **kl** – INTEGER

k_l, the number of subdiagonals within the band of the matrix A.

Constraint: **kl** ≥ 0.

3: **ku** – INTEGER

k_u, the number of superdiagonals within the band of the matrix A.

Constraint: **ku** ≥ 0.

4: **ab**$(ldab,:)$ – REAL (KIND=nag_wp) array

The first dimension of the array **ab** must be at least $2 \times \mathbf{kl} + \mathbf{ku} + 1$.

The second dimension of the array **ab** must be at least $\max(1, \mathbf{n})$.

The LU factorization of A, as returned by nag_lapack_dgbtrf (f07bd).

5: **ipiv**$(:)$ – INTEGER array

The dimension of the array **ipiv** must be at least $\max(1, \mathbf{n})$

The pivot indices, as returned by nag_lapack_dgbtrf (f07bd).

6: **b**$(ldb,:)$ – REAL (KIND=nag_wp) array

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$.

The second dimension of the array **b** must be at least $\max(1, \mathbf{nrhs\_p})$.

The n by r right-hand side matrix B.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the second dimension of the array **ab**.

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

2: **nrhs_p** – INTEGER

Default: the second dimension of the array **b**.

r, the number of right-hand sides.

Constraint: $\mathbf{nrhs\_p} \geq 0$.

5.3 Output Parameters

1: **b**$(ldb,:)$ – REAL (KIND=nag_wp) array

The first dimension of the array **b** will be $\max(1, \mathbf{n})$.

The second dimension of the array **b** will be $\max(1, \mathbf{nrhs\_p})$.

The n by r solution matrix X.

2: **info** – INTEGER

$\mathbf{info} = 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

$\mathbf{info} < 0$

If $\mathbf{info} = -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

For each right-hand side vector b, the computed solution x is the exact solution of a perturbed system of equations $(A + E)x = b$, where

$$|E| \leq c(k)\epsilon P|L||U|,$$

$c(k)$ is a modest linear function of $k = k_l + k_u + 1$, and ϵ is the ***machine precision***. This assumes $k \ll n$.

If $\hat{x}$ is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x - \hat{x}\|_\infty}{\|x\|_\infty} \leq c(k)\,\text{cond}(A, x)\epsilon$$

where $\text{cond}(A, x) = \left\| |A^{-1}||A||x| \right\|_\infty / \|x\|_\infty \leq \text{cond}(A) = \left\| |A^{-1}||A| \right\|_\infty \leq \kappa_\infty(A)$.

Note that $\text{cond}(A, x)$ can be much smaller than $\text{cond}(A)$, and $\text{cond}(A^{\text{T}})$ can be much larger (or smaller) than $\text{cond}(A)$.

Forward and backward error bounds can be computed by calling nag_lapack_dgbrfs (f07bh), and an estimate for $\kappa_\infty(A)$ can be obtained by calling nag_lapack_dgbcon (f07bg) with **norm_p** = 'I'.

8 Further Comments

The total number of floating-point operations is approximately $2n(2k_l + k_u)r$, assuming $n \gg k_l$ and $n \gg k_u$.

This function may be followed by a call to nag_lapack_dgbrfs (f07bh) to refine the solution and return an error estimate.

The complex analogue of this function is nag_lapack_zgbtrs (f07bs).

9 Example

This example solves the system of equations $AX = B$, where

$$A = \begin{pmatrix} -0.23 & 2.54 & -3.66 & 0.00 \\ -6.98 & 2.46 & -2.73 & -2.13 \\ 0.00 & 2.56 & 2.46 & 4.07 \\ 0.00 & 0.00 & -4.78 & -3.82 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 4.42 & -36.01 \\ 27.13 & -31.67 \\ -6.14 & -1.16 \\ 10.50 & -25.82 \end{pmatrix}.$$

Here A is nonsymmetric and is treated as a band matrix, which must first be factorized by nag_lapack_dgbtrf (f07bd).

9.1 Program Text

```
     function f07be_example

fprintf('f07be example results\n\n');

m  = nag_int(4);
kl = nag_int(1);
ku = nag_int(2);
ab = [ 0,     0,     0,     0;
       0,     0,    -3.66, -2.13;
       0,     2.54, -2.73,  4.07;
      -0.23,  2.46,  2.46, -3.82;
      -6.98,  2.56, -4.78,  0];
b  = [ 4.42, -36.01;
      27.13, -31.67;
      -6.14, -1.16;
      10.50, -25.82];
% Factorize A
[abf, ipiv, info] = f07bd( ...
                           m, kl, ku, ab);

% Compute Solution
trans = 'N';
```

```
[x, info] = f07be( ...
                      trans, kl, ku, abf, ipiv, b);

disp('Solution(s)');
disp(x);
```

9.2 Program Results

```
     f07be example results

Solution(s)
   -2.0000    1.0000
    3.0000   -4.0000
    1.0000    7.0000
   -4.0000   -2.0000
```