NAG Toolbox

nag_lapack_zgetrf (f07ar)

1 Purpose

nag_lapack_zgetrf (f07ar) computes the LU factorization of a complex m by n matrix.

2 Syntax

```
[a, ipiv, info] = nag_lapack_zgetrf(a, 'm', m, 'n', n)
[a, ipiv, info] = f07ar(a, 'm', m, 'n', n)
```

3 Description

nag_lapack_zgetrf (f07ar) forms the LU factorization of a complex m by n matrix A as $A = PLU$, where P is a permutation matrix, L is lower triangular with unit diagonal elements (lower trapezoidal if $m > n$) and U is upper triangular (upper trapezoidal if $m < n$). Usually A is square ($m = n$), and both L and U are triangular. The function uses partial pivoting, with row interchanges.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **a**(lda, :) – COMPLEX (KIND=nag_wp) array

The first dimension of the array **a** must be at least max(1, **m**).

The second dimension of the array **a** must be at least max(1, **n**).

The m by n matrix A.

5.2 Optional Input Parameters

1: **m** – INTEGER

Default: the first dimension of the array **a**.

m, the number of rows of the matrix A.

Constraint: **m** ≥ 0.

2: **n** – INTEGER

Default: the second dimension of the array **a**.

n, the number of columns of the matrix A.

Constraint: **n** ≥ 0.

5.3 Output Parameters

1: **a**(lda, :) – COMPLEX (KIND=nag_wp) array

The first dimension of the array **a** will be max(1, **m**).

The second dimension of the array **a** will be $\max(1, \mathbf{n})$.

The factors L and U from the factorization $A = PLU$; the unit diagonal elements of L are not stored.

2: **ipiv**(**min**(**m**, **n**)) – INTEGER array

The pivot indices that define the permutation matrix. At the ith step, if **ipiv**$(i) > i$ then row i of the matrix A was interchanged with row **ipiv**(i), for $i = 1, 2, \ldots, \min(m, n)$. **ipiv**$(i) \leq i$ indicates that, at the ith step, a row interchange was not required.

3: **info** – INTEGER

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info < 0

If **info** $= -i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

info > 0 (*warning*)

Element ⟨*value*⟩ of the diagonal is exactly zero. The factorization has been completed, but the factor U is exactly singular, and division by zero will occur if it is used to solve a system of equations.

7 Accuracy

The computed factors L and U are the exact factors of a perturbed matrix $A + E$, where

$$|E| \leq c(\min(m, n))\epsilon P|L||U|,$$

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

8 Further Comments

The total number of real floating-point operations is approximately $\frac{8}{3}n^3$ if $m = n$ (the usual case), $\frac{4}{3}n^2(3m - n)$ if $m > n$ and $\frac{4}{3}m^2(3n - m)$ if $m < n$.

A call to this function with $m = n$ may be followed by calls to the functions:

nag_lapack_zgetrs (f07as) to solve $AX = B$, $A^{\mathrm{T}}X = B$ or $A^{\mathrm{H}}X = B$;

nag_lapack_zgecon (f07au) to estimate the condition number of A;

nag_lapack_zgetri (f07aw) to compute the inverse of A.

The real analogue of this function is nag_lapack_dgetrf (f07ad).

9 Example

This example computes the LU factorization of the matrix A, where

$$A = \begin{pmatrix} -1.34 + 2.55i & 0.28 + 3.17i & -6.39 - 2.20i & 0.72 - 0.92i \\ -0.17 - 1.41i & 3.31 - 0.15i & -0.15 + 1.34i & 1.29 + 1.38i \\ -3.29 - 2.39i & -1.91 + 4.42i & -0.14 - 1.35i & 1.72 + 1.35i \\ 2.41 + 0.39i & -0.56 + 1.47i & -0.83 - 0.69i & -1.96 + 0.67i \end{pmatrix}.$$

9.1 Program Text

```
     function f07ar_example

fprintf('f07ar example results\n\n');

a = [-1.34 + 2.55i,  0.28 + 3.17i, -6.39 - 2.20i,  0.72 - 0.92i;
     -0.17 - 1.41i,  3.31 - 0.15i, -0.15 + 1.34i,  1.29 + 1.38i;
     -3.29 - 2.39i, -1.91 + 4.42i, -0.14 - 1.35i,  1.72 + 1.35i;
      2.41 + 0.39i, -0.56 + 1.47i, -0.83 - 0.69i, -1.96 + 0.67i];

[LU, ipiv, info] = f07ar(a);

disp('Details of factorization');
disp(LU);
disp('Pivot indices');
disp(double(ipiv'));
```

9.2 Program Results

```
     f07ar example results

Details of factorization
  -3.2900 - 2.3900i  -1.9100 + 4.4200i  -0.1400 - 1.3500i   1.7200 + 1.3500i
   0.2376 + 0.2560i   4.8952 - 0.7114i  -0.4623 + 1.6966i   1.2269 + 0.6190i
  -0.1020 - 0.7010i  -0.6691 + 0.3689i  -5.1414 - 1.1300i   0.9983 + 0.3850i
  -0.5359 + 0.2707i  -0.2040 + 0.8601i   0.0082 + 0.1211i   0.1482 - 0.1252i

Pivot indices
     3     2     3     4
```