

NAG Toolbox

nag_lapack_dgecon (f07ag)

1 Purpose

nag_lapack_dgecon (f07ag) estimates the condition number of a real matrix A , where A has been factorized by nag_lapack_dgetrf (f07ad).

2 Syntax

```
[rcond, info] = nag_lapack_dgecon(norm_p, a, anorm, 'n', n)
[rcond, info] = f07ag(norm_p, a, anorm, 'n', n)
```

3 Description

nag_lapack_dgecon (f07ag) estimates the condition number of a real matrix A , in either the 1-norm or the ∞ -norm:

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1 \quad \text{or} \quad \kappa_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty.$$

Note that $\kappa_\infty(A) = \kappa_1(A^T)$.

Because the condition number is infinite if A is singular, the function actually returns an estimate of the **reciprocal** of the condition number.

The function should be preceded by a computation of $\|A\|_1$ or $\|A\|_\infty$, and a call to nag_lapack_dgetrf (f07ad) to compute the LU factorization of A . The function then uses Higham's implementation of Hager's method (see Higham (1988)) to estimate $\|A^{-1}\|_1$ or $\|A^{-1}\|_\infty$.

4 References

Higham N J (1988) FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

5 Parameters

5.1 Compulsory Input Parameters

1: **norm_p** – CHARACTER(1)

Indicates whether $\kappa_1(A)$ or $\kappa_\infty(A)$ is estimated.

norm_p = '1' or 'O'

$\kappa_1(A)$ is estimated.

norm_p = 'I'

$\kappa_\infty(A)$ is estimated.

Constraint: **norm_p** = '1', 'O' or 'I'.

2: **a(lda,:)** – REAL (KIND=nag_wp) array

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$.

The second dimension of the array **a** must be at least $\max(1, \mathbf{n})$.

The LU factorization of A , as returned by nag_lapack_dgetrf (f07ad).

3: **anorm** – REAL (KIND=nag_wp)

If **norm_p** = '1' or 'O', the 1-norm of the **original** matrix A .

If **norm_p** = 'I', the ∞ -norm of the **original** matrix A .

anorm must be computed either **before** calling nag_lapack_dgetrf (f07ad) or else from a **copy** of the original matrix A (see Section 10).

Constraint: **anorm** ≥ 0.0 .

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the array **a** and the second dimension of the array **a**.

n , the order of the matrix A .

Constraint: **n** ≥ 0 .

5.3 Output Parameters

1: **rcond** – REAL (KIND=nag_wp)

An estimate of the reciprocal of the condition number of A . **rcond** is set to zero if exact singularity is detected or the estimate underflows. If **rcond** is less than *machine precision*, A is singular to working precision.

2: **info** – INTEGER

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info < 0

If **info** = $-i$, argument i had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

The computed estimate **rcond** is never less than the true value ρ , and in practice is nearly always less than 10ρ , although examples can be constructed where **rcond** is much larger.

8 Further Comments

A call to nag_lapack_dgecon (f07ag) involves solving a number of systems of linear equations of the form $Ax = b$ or $A^T x = b$; the number is usually 4 or 5 and never more than 11. Each solution involves approximately $2n^2$ floating-point operations but takes considerably longer than a call to nag_lapack_dgetrs (f07ae) with one right-hand side, because extra care is taken to avoid overflow when A is approximately singular.

The complex analogue of this function is nag_lapack_zgecon (f07au).

9 Example

This example estimates the condition number in the 1-norm of the matrix A , where

$$A = \begin{pmatrix} 1.80 & 2.88 & 2.05 & -0.89 \\ 5.25 & -2.95 & -0.95 & -3.80 \\ 1.58 & -2.69 & -2.90 & -1.04 \\ -1.11 & -0.66 & -0.59 & 0.80 \end{pmatrix}.$$

Here A is nonsymmetric and must first be factorized by `nag_lapack_dgetrf` (f07ad). The true condition number in the 1-norm is 152.16.

9.1 Program Text

```
function f07ag_example

fprintf('f07ag example results\n\n');

a = [ 1.80,  2.88,  2.05, -0.89;
      5.25, -2.95, -0.95, -3.80;
      1.58, -2.69, -2.90, -1.04;
     -1.11, -0.66, -0.59,  0.80];

anorm = norm(a, 1);

% Factorize A
[a, ipiv, info] = f07ad(a);

% Estimate condition number
norm_p = '1';
[rcond, info] = f07ag(norm_p, a, anorm);

if rcond > x02aj
    fprintf('Estimate of condition number = %10.2e\n', 1/rcond);
else
    fprintf('A is singular to working precision\n');
end
```

9.2 Program Results

```
f07ag example results

Estimate of condition number = 1.52e+02
```
