NAG Toolbox

nag_linsys_real_posdef_tridiag_solve (f04bg)

1 Purpose

nag_linsys_real_posdef_tridiag_solve (f04bg) computes the solution to a real system of linear equations $AX = B$, where A is an n by n symmetric positive definite tridiagonal matrix and X and B are n by r matrices. An estimate of the condition number of A and an error bound for the computed solution are also returned.

2 Syntax

```
[d, e, b, rcond, errbnd, ifail] = nag_linsys_real_posdef_tridiag_solve(d, e, b,
'n', n, 'nrhs_p', nrhs_p)
```

```
[d, e, b, rcond, errbnd, ifail] = f04bg(d, e, b, 'n', n, 'nrhs_p', nrhs_p)
```

3 Description

A is factorized as $A = LDL^{\mathrm{T}}$, where L is a unit lower bidiagonal matrix and D is diagonal, and the factored form of A is then used to solve the system of equations.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia http://www.netlib.org/lapack/lug

Higham N J (2002) *Accuracy and Stability of Numerical Algorithms* (2nd Edition) SIAM, Philadelphia

5 Parameters

5.1 Compulsory Input Parameters

1: **d**(:) – REAL (KIND=nag_wp) array

The dimension of the array **d** must be at least $\max(1, \mathbf{n})$

Must contain the n diagonal elements of the tridiagonal matrix A.

2: **e**(:) – REAL (KIND=nag_wp) array

The dimension of the array **e** must be at least $\max(1, \mathbf{n} - 1)$

Must contain the $(n - 1)$ subdiagonal elements of the tridiagonal matrix A.

3: **b**(ldb, :) – REAL (KIND=nag_wp) array

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$.

The second dimension of the array **b** must be at least $\max(1, \mathbf{nrhs\_p})$.

The n by r matrix of right-hand sides B.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the array **b**.

The number of linear equations n, i.e., the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

2: **nrhs_p** – INTEGER

Default: the second dimension of the array **b**.

The number of right-hand sides r, i.e., the number of columns of the matrix B.

Constraint: $\mathbf{nrhs\_p} \geq 0$.

5.3 Output Parameters

1: **d**$(:)$ – REAL (KIND=nag_wp) array

The dimension of the array **d** will be $\max(1, \mathbf{n})$

If **ifail** $= 0$ or $\mathbf{n} + 1$, **d** stores the n diagonal elements of the diagonal matrix D from the LDL^{T} factorization of A.

2: **e**$(:)$ – REAL (KIND=nag_wp) array

The dimension of the array **e** will be $\max(1, \mathbf{n} - 1)$

If **ifail** $= 0$ or $\mathbf{n} + 1$, **e** stores the $(n - 1)$ subdiagonal elements of the unit lower bidiagonal matrix L from the LDL^{T} factorization of A. (**e** can also be regarded as the superdiagonal of the unit upper bidiagonal factor U from the $U^{\mathrm{T}}DU$ factorization of A.)

3: **b**$(ldb, :)$ – REAL (KIND=nag_wp) array

The first dimension of the array **b** will be $\max(1, \mathbf{n})$.

The second dimension of the array **b** will be $\max(1, \mathbf{nrhs\_p})$.

If **ifail** $= 0$ or $\mathbf{n} + 1$, the n by r solution matrix X.

4: **rcond** – REAL (KIND=nag_wp)

If **ifail** $= 0$ or $\mathbf{n} + 1$, an estimate of the reciprocal of the condition number of the matrix A, computed as $\mathbf{rcond} = 1/\left(\|A\|_1 \|A^{-1}\|_1\right)$.

5: **errbnd** – REAL (KIND=nag_wp)

If **ifail** $= 0$ or $\mathbf{n} + 1$, an estimate of the forward error bound for a computed solution $\hat{x}$, such that $\|\hat{x} - x\|_1 / \|x\|_1 \leq \mathbf{errbnd}$, where $\hat{x}$ is a column of the computed solution returned in the array **b** and x is the corresponding column of the exact solution X. If **rcond** is less than ***machine precision***, then **errbnd** is returned as unity.

6: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail > 0 and **ifail** $\leq \mathbf{n}$

The principal minor of order $\langle value \rangle$ of the matrix A is not positive definite. The factorization has not been completed and the solution could not be computed.

ifail $= \mathbf{n} + 1$ (*warning*)

A solution has been computed, but **rcond** is less than ***machine precision*** so that the matrix A is numerically singular.

ifail $= -1$

Constraint: $\mathbf{n} \geq 0$.

ifail $= -2$

Constraint: $\mathbf{nrhs\_p} \geq 0$.

ifail $= -6$

Constraint: $ldb \geq \max(1, \mathbf{n})$.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

*The double allocatable memory required is **n**. In this case the factorization and the solution X have been computed, but **rcond** and **errbnd** have not been computed.*

7 Accuracy

The computed solution for a single right-hand side, $\hat{x}$, satisfies an equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_1 = O(\epsilon)\|A\|_1$$

and ϵ is the ***machine precision***. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x} - x\|_1}{\|x\|_1} \leq \kappa(A)\frac{\|E\|_1}{\|A\|_1},$$

where $\kappa(A) = \left\|A^{-1}\right\|_1\|A\|_1$, the condition number of A with respect to the solution of the linear equations. nag_linsys_real_posdef_tridiag_solve (f04bg) uses the approximation $\|E\|_1 = \epsilon\|A\|_1$ to estimate **errbnd**. See Section 4.4 of Anderson *et al.* (1999) for further details.

8 Further Comments

The total number of floating-point operations required to solve the equations $AX = B$ is proportional to nr. The condition number estimation requires $O(n)$ floating-point operations.

See Section 15.3 of Higham (2002) for further details on computing the condition number of tridiagonal matrices.

The complex analogue of nag_linsys_real_posdef_tridiag_solve (f04bg) is nag_linsys_complex_posdef_tridiag_solve (f04cg).

9 Example

This example solves the equations

$$AX = B,$$

where A is the symmetric positive definite tridiagonal matrix

$$A = \begin{pmatrix} 4.0 & -2.0 & 0 & 0 & 0 \\ -2.0 & 10.0 & -6.0 & 0 & 0 \\ 0 & -6.0 & 29.0 & 15.0 & 0 \\ 0 & 0 & 15.0 & 25.0 & 8.0 \\ 0 & 0 & 0 & 8.0 & 5.0 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 6.0 & 10.0 \\ 9.0 & 4.0 \\ 2.0 & 9.0 \\ 14.0 & 65.0 \\ 7.0 & 23.0 \end{pmatrix}.$$

An estimate of the condition number of A and an approximate error bound for the computed solutions are also printed.

9.1 Program Text

```
function f04bg_example

fprintf('f04bg example results\n\n');

% Solve Ax = b for symmetric tridiagonal A with error and conditioning
d = [ 4    10     29     25     5];
e = [-2    -6     15      8];
b = [ 6, 10;
      9,  4;
      2,  9;
     14, 65;
      7, 23];

[d, e, x, rcond, errbnd, ifail] = ...
  f04bg(d, e, b);

disp('Solution');
disp(x);
disp('Estimate of condition number');
fprintf('%10.1f\n\n',1/rcond);
disp('Estimate of error bound for computed solutions');
fprintf('%10.1e\n\n',errbnd);
```

9.2 Program Results

```
f04bg example results

Solution
    2.5000    2.0000
    2.0000   -1.0000
    1.0000   -3.0000
   -1.0000    6.0000
    3.0000   -5.0000

Estimate of condition number
     105.0

Estimate of error bound for computed solutions
   1.2e-14
```