NAG Toolbox

nag_fit_1dspline_integ (e02bd)

1 Purpose

nag_fit_1dspline_integ (e02bd) computes the definite integral of a cubic spline from its B-spline representation.

2 Syntax

```
[dint, ifail] = nag_fit_1dspline_integ(lamda, c, 'ncap7', ncap7)
[dint, ifail] = e02bd(lamda, c, 'ncap7', ncap7)
```

3 Description

nag_fit_1dspline_integ (e02bd) computes the definite integral of the cubic spline $s(x)$ between the limits $x = a$ and $x = b$, where a and b are respectively the lower and upper limits of the range over which $s(x)$ is defined. It is assumed that $s(x)$ is represented in terms of its B-spline coefficients c_i, for $i = 1, 2, \ldots, \bar{n} + 3$ and (augmented) ordered knot set λ_i, for $i = 1, 2, \ldots, \bar{n} + 7$, with $\lambda_i = a$, for $i = 1, 2, 3, 4$ and $\lambda_i = b$, for $i = \bar{n} + 4, \ldots, \bar{n} + 7$, (see nag_fit_1dspline_knots (e02ba)), i.e.,

$$s(x) = \sum_{i=1}^{q} c_i N_i(x).$$

Here $q = \bar{n} + 3$, $\bar{n}$ is the number of intervals of the spline and $N_i(x)$ denotes the normalized B-spline of degree 3 (order 4) defined upon the knots $\lambda_i, \lambda_{i+1}, \ldots, \lambda_{i+4}$.

The method employed uses the formula given in Section 3 of Cox (1975).

nag_fit_1dspline_integ (e02bd) can be used to determine the definite integrals of cubic spline fits and interpolants produced by nag_fit_1dspline_knots (e02ba).

4 References

Cox M G (1975) An algorithm for spline interpolation *J. Inst. Math. Appl.* **15** 95–108

5 Parameters

5.1 Compulsory Input Parameters

1: **lamda**(**ncap7**) – REAL (KIND=nag_wp) array

lamda(j) must be set to the value of the jth member of the complete set of knots, λ_j, for $j = 1, 2, \ldots, \bar{n} + 7$.

Constraint: the **lamda**(j) must be in nondecreasing order with **lamda**$(\mathbf{ncap7} - 3) >$ **lamda**(4) and satisfy **lamda**$(1) =$ **lamda**$(2) =$ **lamda**$(3) =$ **lamda**(4) and **lamda**$(\mathbf{ncap7} - 3) =$ **lamda**$(\mathbf{ncap7} - 2) =$ **lamda**$(\mathbf{ncap7} - 1) =$ **lamda**$(\mathbf{ncap7})$.

2: **c**(**ncap7**) – REAL (KIND=nag_wp) array

The coefficient c_i of the B-spline $N_i(x)$, for $i = 1, 2, \ldots, \bar{n} + 3$. The remaining elements of the array are not referenced.

5.2 Optional Input Parameters

1: **ncap7** – INTEGER

Default: the dimension of the arrays **lamda**, **c**. (An error is raised if these dimensions are not equal.)

$\bar{n} + 7$, where $\bar{n}$ is the number of intervals of the spline (which is one greater than the number of interior knots, i.e., the knots strictly within the range a to b) over which the spline is defined.

Constraint: **ncap7** ≥ 8.

5.3 Output Parameters

1: **dint** – REAL (KIND=nag_wp)

The value of the definite integral of $s(x)$ between the limits $x = a$ and $x = b$, where $a = \lambda_4$ and $b = \lambda_{\bar{n}+4}$.

2: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

ncap7 < 8, i.e., the number of intervals is not positive.

ifail $= 2$

At least one of the following restrictions on the knots is violated:

lamda(**ncap7** $- 3) >$ **lamda**(4),

lamda$(j) \geq$ **lamda**$(j - 1)$,

for $j = 2, 3, \ldots,$ **ncap7**, with equality in the cases $j = 2, 3, 4,$ **ncap7** $- 2,$ **ncap7** $- 1$, and **ncap7**.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

The rounding errors are such that the computed value of the integral is exact for a slightly perturbed set of B-spline coefficients c_i differing in a relative sense from those supplied by no more than $2.2 \times (\bar{n} + 3) \times$ ***machine precision***.

8 Further Comments

The time taken is approximately proportional to $\bar{n} + 7$.

9 Example

This example determines the definite integral over the interval $0 \leq x \leq 6$ of a cubic spline having 6 interior knots at the positions $\lambda = 1$, 3, 3, 3, 4, 4, the 8 additional knots 0, 0, 0, 0, 6, 6, 6, 6, and the 10 B-spline coefficients 10, 12, 13, 15, 22, 26, 24, 18, 14, 12.

The input data items (using the notation of Section 5) comprise the following values in the order indicated:

$\bar{n}$
lamda(j), for $j = 1, 2, \ldots,$ **ncap7**
c(j), for $j = 1, 2, \ldots,$ **ncap7** $- 3$

9.1 Program Text

```
function e02bd_example

fprintf('e02bd example results\n\n');

knots = [ 1  3  3  3  4  4];
ncap = size(knots,2) + 1;
ncap7 = ncap + 7;

lamda = zeros(ncap7,1);
lamda(5:ncap+3) = knots;
lamda(ncap+4:ncap7) = 6;

% B-spline coefficients
c = zeros(ncap7,1);
c(1:ncap+3) = [10  12  13  15  22  26  24  18  14  12];

[dint, ifail] = e02bd( ...
                       lamda, c);

fprintf('Definite integral = %10.3e\n',dint);
```

9.2 Program Results

```
e02bd example results

Definite integral =  1.000e+02
```