NAG Toolbox

nag_quad_md_numth_coeff_2prime (d01gz)

1 Purpose

nag_quad_md_numth_coeff_2prime (d01gz) calculates the optimal coefficients for use by nag_quad_md_numth (d01gc) and nag_quad_md_numth_vec (d01gd), when the number of points is the product of two primes.

2 Syntax

```
[vk, ifail] = nag_quad_md_numth_coeff_2prime(ndim, np1, np2)
[vk, ifail] = d01gz(ndim, np1, np2)
```

3 Description

Korobov (1963) gives a procedure for calculating optimal coefficients for p-point integration over the n-cube $[0,1]^n$, when the number of points is

$$p = p_1 p_2 \tag{1}$$

where p_1 and p_2 are distinct prime numbers.

The advantage of this procedure is that if p_1 is chosen to be the nearest prime integer to p_2^2, then the number of elementary operations required to compute the rule is of the order of $p^{4/3}$ which grows less rapidly than the number of operations required by nag_quad_md_numth_coeff_prime (d01gy). The associated error is likely to be larger although it may be the only practical alternative for high values of p.

4 References

Korobov N M (1963) *Number Theoretic Methods in Approximate Analysis* Fizmatgiz, Moscow

5 Parameters

5.1 Compulsory Input Parameters

1: **ndim** – INTEGER

n, the number of dimensions of the integral.

Constraint: **ndim** ≥ 1.

2: **np1** – INTEGER

The larger prime factor p_1 of the number of points in the integration rule.

Constraint: **np1** must be a prime number ≥ 5.

3: **np2** – INTEGER

The smaller prime factor p_2 of the number of points in the integration rule. For maximum efficiency, p_2^2 should be close to p_1.

Constraint: **np2** must be a prime number such that **np1** $>$ **np2** ≥ 2.

5.2 Optional Input Parameters

None.

5.3 Output Parameters

1: **vk**(**ndim**) – REAL (KIND=nag_wp) array

The n optimal coefficients.

2: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, **ndim** < 1.

ifail $= 2$

On entry, **np1** < 5,
or **np2** < 2,
or **np1** $\leq$ **np2**.

ifail $= 3$

The value **np1** $\times$ **np2** exceeds the largest integer representable on the machine, and hence the optimal coefficients could not be used in a valid call of nag_quad_md_numth (d01gc) or nag_quad_md_numth_vec (d01gd).

ifail $= 4$

On entry, **np1** is not a prime number.

ifail $= 5$

On entry, **np2** is not a prime number.

ifail $= 6$ (*warning*)

The precision of the machine is insufficient to perform the computation exactly. Try smaller values of **np1** or **np2**, or use an implementation with higher precision.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

The optimal coefficients are returned as exact integers (though stored in a double array).

8 Further Comments

The time taken by nag_quad_md_numth_coeff_2prime (d01gz) grows at least as fast as $(p_1 p_2)^{4/3}$. (See Section 3.)

9 Example

This example calculates the Korobov optimal coefficients where the number of dimensons is 4 and the number of points is the product of the two prime numbers, 89 and 11.

9.1 Program Text

```
     function d01gz_example

fprintf('d01gz example results\n\n');

ndim = nag_int(4);
np1 = nag_int(89);
np2 = nag_int(11);

[vk, ifail] = d01gz(ndim, np1, np2);

fprintf('Optimal coefficients:');
fprintf('%6d',vk);
fprintf('\n');
```

9.2 Program Results

```
     d01gz example results

Optimal coefficients:     1   102   614   951
```