NAG Toolbox

nag_mv_rot_promax (g03bd)

1 Purpose

nag_mv_rot_promax (g03bd) calculates a ProMax rotation, given information following an orthogonal rotation.

2 Syntax

```
[fp, r, phi, fs, ifail] = nag_mv_rot_promax(stand, x, ro, power, 'n', n, 'm', m)
[fp, r, phi, fs, ifail] = g03bd(stand, x, ro, power, 'n', n, 'm', m)
```

3 Description

Let X and Y denote n by m matrices each representing a set of n points in an m-dimensional space. The X matrix is a matrix of loadings as returned by nag_mv_rot_orthomax (g03ba), that is following an orthogonal rotation of a loadings matrix Z. The target matrix Y is calculated as a power transformation of X that preserves the sign of the loadings. Let X_{ij} and Y_{ij} denote the (i,j)th element of matrices X and Y. Given a value greater than one for the exponent p:

$$Y_{ij} = \delta_{ij}\left\|X_{ij}\right\|^p,$$

for

$$i = 1,2,\dots,n;$$

$$j = 1,2,\dots,m;$$

$$\delta_{ij} = \begin{cases} -1, & \text{if } X_{ij} < 0; \\ 1, & \text{otherwise.} \end{cases}$$

The above power transformation tends to increase the difference between high and low values of loadings and is intended to increase the interpretability of a solution.

In the second step a solution of:

$$XW = Y, \quad X,Y \in \mathbb{R}^{n\times m},\, W \in \mathbb{R}^{m\times m},$$

is found for W in the least squares sense by use of singular value decomposition of the orthogonal loadings X. The ProMax rotation matrix R is then given by

$$R = OW\tilde{W}, \quad O,\, \tilde{W} \in \mathbb{R}^{m\times m},$$

where O is the rotation matrix from an orthogonal transformation, and $\tilde{W}$ is a matrix with the square root of diagonal elements of $\left(W^{\mathrm{T}}W\right)^{-1}$ on its diagonal and zeros elsewhere.

The ProMax factor pattern matrix P is given by

$$P = XW\tilde{W}, \quad P \in \mathbb{R}^{n\times m};$$

the inter-factor correlations Φ are given by

$$\Phi = \left(Q^{\mathrm{T}}Q\right)^{-1}, \quad \Phi \in \mathbb{R}^{m\times m};$$

where $Q = W\tilde{W}$; and the factor structure S is given by

$$S = P\Phi, \quad S \in \mathbb{R}^{n\times m}.$$

Optionally, the rows of target matrix Y can be scaled by the communalities of loadings.

4 References

None.

5 Parameters

5.1 Compulsory Input Parameters

1: **stand** – CHARACTER(1)

Indicates how loadings are normalized.

stand = 'S'
Rows of Y are (Kaiser) normalized by the communalities of the loadings.

stand = 'U'
Rows are not normalized.

Constraint: **stand** = 'U' or 'S'.

2: $\mathbf{x}(ldx, \mathbf{m})$ – REAL (KIND=nag_wp) array

ldx, the first dimension of the array, must satisfy the constraint $ldx \geq \mathbf{n}$.

The loadings matrix following an orthogonal rotation, X.

3: $\mathbf{ro}(ldro, \mathbf{m})$ – REAL (KIND=nag_wp) array

$ldro$, the first dimension of the array, must satisfy the constraint $ldro \geq \mathbf{m}$.

The orthogonal rotation matrix, O.

4: **power** – REAL (KIND=nag_wp)

p, the value of exponent.

Constraint: **power** > 1.0.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the array **x**.

n, the number of points.

Constraint: $\mathbf{n} \geq \mathbf{m}$.

2: **m** – INTEGER

Default: the first dimension of the array **ro** and the second dimension of the arrays **x**, **ro**. (An error is raised if these dimensions are not equal.)

m, the number of dimensions.

Constraint: $\mathbf{m} \geq 1$.

5.3 Output Parameters

1: $\mathbf{fp}(ldfp, \mathbf{m})$ – REAL (KIND=nag_wp) array

The factor pattern matrix, P.

2: $\mathbf{r}(ldr, \mathbf{m})$ – REAL (KIND=nag_wp) array

The ProMax rotation matrix, R.

3: **phi**$(ldphi, \mathbf{m})$ – REAL (KIND=nag_wp) array

The matrix of inter-factor correlations, Φ.

4: **fs**$(ldfs, \mathbf{m})$ – REAL (KIND=nag_wp) array

The factor structure matrix, S.

5: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

Constraint: $\mathbf{m} \geq 1$.

Constraint: **power** > 1.0.

Constraint: **stand** $=$ 'U' or 'S'.

ifail $= 2$

Constraint: $ldfp \geq \mathbf{n}$.

Constraint: $ldfs \geq \mathbf{n}$.

Constraint: $ldphi \geq \mathbf{m}$.

Constraint: $ldro \geq \mathbf{m}$.

Constraint: $ldr \geq \mathbf{m}$.

Constraint: $ldx \geq \mathbf{n}$.

Constraint: $\mathbf{n} \geq \mathbf{m}$.

ifail $= 20$

SVD failed to converge.

ifail $= 100$

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

The calculations are believed to be stable.

8 Further Comments

None.

9 Example

This example reads a loadings matrix and calculates a varimax transformation before calculating P, R and σ for a ProMax rotation.

9.1 Program Text

```
function g03bd_example

fprintf('g03bd example results\n\n');

fl = [0.74215 -0.57806;
      0.71370 -0.55515;
      0.87899 -0.15847;
      0.62533  0.76621;
      0.71447  0.67936];

% Calculate orthogonal rotation
stand = 's';
g     = 1;
[n,m] = size(fl);
nvar  = nag_int(n);

[~, x, ro, iter, ifail] = g03ba( ...
  stand, g, nvar, fl);

% Calculate ProMax rotation

power = 3;
[fp, r, phi, fs, ifail] = g03bd( ...
  stand, x, ro, power);

mtitle = 'Factor pattern';
matrix = 'General';
diag   = ' ';

[ifail] = x04ca( ...
                 matrix, diag, fp, mtitle);

fprintf('\n');
mtitle = 'Promax rotation';
[ifail] = x04ca( ...
                 matrix, diag, r, mtitle);

fprintf('\n');
mtitle = 'Inter-factor correlations';
[ifail] = x04ca( ...
                 matrix, diag, phi, mtitle);

fprintf('\n');
mtitle = 'Factor structure';
[ifail] = x04ca( ...
                 matrix, diag, fs, mtitle);
```

9.2 Program Results

```
g03bd example results

 Factor pattern
              1          2
 1       0.9556    -0.0979
 2       0.9184    -0.0935
 3       0.7605     0.3393
 4      -0.0791     1.0019
```

```
5      0.0480     0.9751

Promax rotation
         1       2
1   0.7380  0.5420
2  -0.7055  0.8653

Inter-factor correlations
         1       2
1   1.0000  0.2019
2   0.2019  1.0000

Factor structure
         1       2
1   0.9358  0.0950
2   0.8995  0.0919
3   0.8290  0.4928
4   0.1232  0.9860
5   0.2448  0.9848
```