### **NAG Toolbox**

# nag sum convcorr real nowork (c06ek)

### 1 Purpose

nag\_sum\_convcorr\_real\_nowork (c06ek) calculates the circular convolution or correlation of two real vectors of period n. (No extra workspace is required.)

**Note**: This function is scheduled to be withdrawn, please see c06ek in Advice on Replacement Calls for Withdrawn/Superseded Routines..

# 2 Syntax

### 3 Description

nag\_sum\_convcorr\_real\_nowork (c06ek) computes:

if job = 1, the discrete **convolution** of x and y, defined by

$$z_k = \sum_{j=0}^{n-1} x_j y_{k-j} = \sum_{j=0}^{n-1} x_{k-j} y_j;$$

if job = 2, the discrete **correlation** of x and y defined by

$$w_k = \sum_{j=0}^{n-1} x_j y_{k+j}.$$

Here x and y are real vectors, assumed to be periodic, with period n, i.e.,  $x_j = x_{j\pm n} = x_{j\pm 2n} = \dots$ ; z and w are then also periodic with period n.

**Note:** this usage of the terms 'convolution' and 'correlation' is taken from Brigham (1974). The term 'convolution' is sometimes used to denote both these computations.

If  $\hat{x}$ ,  $\hat{y}$ ,  $\hat{z}$  and  $\hat{w}$  are the discrete Fourier transforms of these sequences, i.e.,

$$\hat{x}_k = \frac{1}{\sqrt{n}} \sum_{j=0}^{n-1} x_j \times \exp\left(-i\frac{2\pi jk}{n}\right), \text{ etc.},$$

then  $\hat{z}_k = \sqrt{n}.\hat{x}_k\hat{y}_k$  and  $\hat{w}_k = \sqrt{n}.\bar{x}_k\hat{y}_k$  (the bar denoting complex conjugate).

This function calls the same auxiliary functions as  $nag_sum_fft_real_1d_nowork$  (c06ea) and  $nag_sum_fft_hermitian_1d_nowork$  (c06eb) to compute discrete Fourier transforms, and there are some restrictions on the value of n.

#### 4 References

Brigham E O (1974) The Fast Fourier Transform Prentice-Hall

Mark 25 c06ek.1

#### 5 Parameters

# 5.1 Compulsory Input Parameters

1: **job** – INTEGER

The computation to be performed.

$$\mathbf{job} = 1$$
 
$$z_k = \sum_{j=0}^{n-1} x_j y_{k-j} \text{ (convolution)};$$

$$\mathbf{job} = 2$$
 
$$w_k = \sum_{j=0}^{n-1} x_j y_{k+j} \text{ (correlation)}.$$

Constraint: job = 1 or 2.

2:  $\mathbf{x}(\mathbf{n}) - \text{REAL (KIND=nag_wp)}$  array

The elements of one period of the vector x. If  $\mathbf{x}$  is declared with bounds  $(0:\mathbf{n}-1)$  in the function from which nag\_sum\_convcorr\_real\_nowork (c06ek) is called, then  $\mathbf{x}(j)$  must contain  $x_j$ , for  $j=0,1,\ldots,n-1$ .

3:  $y(n) - REAL (KIND=nag_wp) array$ 

The elements of one period of the vector y. If  $\mathbf{y}$  is declared with bounds  $(0:\mathbf{n}-1)$  in the function from which nag\_sum\_convcorr\_real\_nowork (c06ek) is called, then  $\mathbf{y}(j)$  must contain  $y_j$ , for  $j=0,1,\ldots,n-1$ .

#### 5.2 Optional Input Parameters

1:  $\mathbf{n} - \text{INTEGER}$ 

Default: the dimension of the arrays  $\mathbf{x}$ ,  $\mathbf{y}$ . (An error is raised if these dimensions are not equal.) n, the number of values in one period of the vectors  $\mathbf{x}$  and  $\mathbf{y}$ .

Constraint:  $\mathbf{n} > 1$ .

#### 5.3 Output Parameters

1:  $\mathbf{x}(\mathbf{n}) - \text{REAL}$  (KIND=nag wp) array

The corresponding elements of the discrete convolution or correlation.

2: y(n) - REAL (KIND=nag wp) array

The discrete Fourier transform of the convolution or correlation returned in the array x; the transform is stored in Hermitian form. If the components of the transform are:

$$\hat{Z}_k = a_k + ib_k \\ \hat{Z}_{n-k} = a_k - ib_k$$
  $k = 0, 1, \dots n/2$ 

where  $b_0$  and  $b_{n/2}$  when n is even then  $\mathbf{x}(k+1)$  holds  $a_k$  and  $\mathbf{x}(n-k+1)$  holds nonzero  $b_k$  (see Section 2.1.2 in the C06 Chapter Introduction).

3: **ifail** – INTEGER

**ifail** = 0 unless the function detects an error (see Section 5).

c06ek.2 Mark 25

# 6 Error Indicators and Warnings

Errors or warnings detected by the function:

#### ifail = 1

At least one of the prime factors of  $\mathbf{n}$  is greater than 19.

#### ifail = 2

**n** has more than 20 prime factors.

ifail = 3

On entry,  $\mathbf{n} < 1$ .

ifail = 4

On entry, **job**  $\neq$  1 or 2.

ifail = -99

An unexpected error has been triggered by this routine. Please contact NAG.

**ifail** = -399

Your licence key may have expired or may not have been installed correctly.

**ifail** = -999

Dynamic memory allocation failed.

### 7 Accuracy

The results should be accurate to within a small multiple of the *machine precision*.

#### 8 Further Comments

The time taken is approximately proportional to  $n \times \log(n)$ , but also depends on the factorization of n. nag\_sum\_convcorr\_real\_nowork (c06ek) is faster if the only prime factors of n are 2, 3 or 5; and fastest of all if n is a power of 2.

On the other hand, nag\_sum\_convcorr\_real\_nowork (c06ek) is particularly slow if n has several unpaired prime factors, i.e., if the 'square-free' part of n has several factors. For such values of n, nag\_sum\_convcorr\_real (c06fk) (which requires additional double workspace) is considerably faster.

# 9 Example

This example reads in the elements of one period of two real vectors x and y, and prints their discrete convolution and correlation (as computed by nag\_sum\_convcorr\_real\_nowork (c06ek)). In realistic computations the number of data values would be much larger.

### 9.1 Program Text

```
function c06ek_example
fprintf('c06ek example results\n\n');
           1;
                 1;
                                         0;
                                                0;
                             1;
                                   0;
x = [1;
                       1;
                                                      01:
y = [0.5; 0.5; 0.5; 0.5;
                            0;
                                   0;
                                         0;
                                                      0];
job = nag_int(1);
[conv, tconv, ifail] = c06ek(job, x, y);
```

Mark 25 c06ek.3

```
job = nag_int(2);
[corr, tcorr, ifail] = c06ek(job, x, y);

result = [conv corr];
disp('Convolution Correlation');
disp(result);
```

# 9.2 Program Results

c06ek example results

Convolution	Correlation	
0.5000	2.0000	
1.0000	1.5000	
1.5000	1.0000	
2.0000	0.5000	
2.0000	0.0000	
1.5000	0.5000	
1.0000	1.0000	
0.5000	1.5000	
0.0000	2.0000	

c06ek.4 (last) Mark 25