# NAG Toolbox

# nag_zeros_quartic_complex (c02an)

## 1 Purpose

nag_zeros_quartic_complex (c02an) determines the roots of a quartic equation with complex coefficients.

## 2 Syntax

```
[zeror, zeroi, errest, ifail] = nag_zeros_quartic_complex(e, a, b, c, d)
[zeror, zeroi, errest, ifail] = c02an(e, a, b, c, d)
```

## 3 Description

nag_zeros_quartic_complex (c02an) attempts to find the roots of the quartic equation

$$ez^4 + az^3 + bz^2 + cz + d = 0,$$

where $e$, $a$, $b$, $c$ and $d$ are complex coefficients with $e \neq 0$. The roots are located by finding the eigenvalues of the associated 4 by 4 (upper Hessenberg) companion matrix $H$ given by

$$H = \begin{pmatrix} 0 & 0 & 0 & -d/e \\ 1 & 0 & 0 & -c/e \\ 0 & 1 & 0 & -b/e \\ 0 & 0 & 1 & -a/e \end{pmatrix}.$$

The eigenvalues are obtained by a call to nag_lapack_zhseqr (f08ps). Further details can be found in Section 9.

To obtain the roots of a cubic equation, nag_zeros_cubic_complex (c02am) can be used.

## 4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

## 5 Parameters

### 5.1 Compulsory Input Parameters

1: **e** – COMPLEX (KIND=nag_wp)

$e$, the coefficient of $z^4$.

*Constraint*: $\mathbf{e} \neq (0.0, 0.0)$.

2: **a** – COMPLEX (KIND=nag_wp)

$a$, the coefficient of $z^3$.

3: **b** – COMPLEX (KIND=nag_wp)

$b$, the coefficient of $z^2$.

4: **c** – COMPLEX (KIND=nag_wp)

$c$, the coefficient of $z$.

5: **d** – COMPLEX (KIND=nag_wp)

$d$, the constant coefficient.

### 5.2 Optional Input Parameters

None.

### 5.3 Output Parameters

1: **zeror**(**4**) – REAL (KIND=nag_wp) array
2: **zeroi**(**4**) – REAL (KIND=nag_wp) array

**zeror**$(i)$ and **zeroi**$(i)$ contain the real and imaginary parts, respectively, of the $i$th root.

3: **errest**(**4**) – REAL (KIND=nag_wp) array

**errest**$(i)$ contains an approximate error estimate for the $i$th root.

4: **ifail** – INTEGER

**ifail** $= 0$ unless the function detects an error (see Section 5).

## 6 Error Indicators and Warnings

Errors or warnings detected by the function:

**ifail** $= 1$

On entry, **e** $= (0.0, 0.0)$.

**ifail** $= 2$

The companion matrix $H$ cannot be formed without overflow.

**ifail** $= 3$

The iterative procedure used to determine the eigenvalues has failed to converge.

**ifail** $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

**ifail** $= -399$

Your licence key may have expired or may not have been installed correctly.

**ifail** $= -999$

Dynamic memory allocation failed.

## 7 Accuracy

If **ifail** $= 0$ on exit, then the $i$th computed root should have approximately $|\log_{10}(\textbf{errest}(i))|$ correct significant digits.

## 8 Further Comments

The method used by the function consists of the following steps, which are performed by functions from LAPACK in Chapter F08.

(a) Form matrix $H$.

(b) Apply a diagonal similarity transformation to $H$ (to give $H'$).

(c) Calculate the eigenvalues and Schur factorization of $H'$.

(d) Calculate the left and right eigenvectors of $H'$.

(e) Estimate reciprocal condition numbers for all the eigenvalues of $H'$.

(f) Calculate approximate error estimates for all the eigenvalues of $H'$ (using the 1-norm).

# 9 Example

This example finds the roots of the quartic equation

$$z^4 + 16iz^2 - (8 - 8i)z - 65 = 0.$$

## 9.1 Program Text

```
     function c02an_example

fprintf('c02an example results\n\n');

e =  complex(1);
a =   -10 + 2i;
b =    48 - 10i;
c =  -100 + 28i;
d =  complex(96);
[zr, zi, errest, ifail] = c02an(e, a, b, c, d);

fprintf('  Roots of quartic    error estimates\n');
for j = 1:4
   if (zi(j)<0)
     fprintf('%8.4f - %7.4fi     %8.2e\n',zr(j),abs(zi(j)),errest(j));
   else
     fprintf('%8.4f - %7.4fi     %8.2e\n',zr(j),abs(zi(j)),errest(j));
   end
end
```

## 9.2 Program Results

```
     c02an example results

  Roots of quartic    error estimates
  3.0000 -  3.0000i     1.55e-14
  1.0000 -  1.0000i     1.18e-14
  2.0000 -  2.0000i     2.14e-14
  4.0000 -  4.0000i     2.02e-14
```