NAG Toolbox

nag_zeros_quartic_real (c02al)

1 Purpose

nag_zeros_quartic_real (c02al) determines the roots of a quartic equation with real coefficients.

2 Syntax

```
[zeror, zeroi, errest, ifail] = nag_zeros_quartic_real(e, a, b, c, d)
[zeror, zeroi, errest, ifail] = c02al(e, a, b, c, d)
```

3 Description

nag_zeros_quartic_real (c02al) attempts to find the roots of the quartic equation

$$ez^4 + az^3 + bz^2 + cz + d = 0,$$

where e, a, b, c and d are real coefficients with $e \neq 0$. The roots are located by finding the eigenvalues of the associated 4 by 4 (upper Hessenberg) companion matrix H given by

$$H = \begin{pmatrix} 0 & 0 & 0 & -d/e \\ 1 & 0 & 0 & -c/e \\ 0 & 1 & 0 & -b/e \\ 0 & 0 & 1 & -a/e \end{pmatrix}.$$

The eigenvalues are obtained by a call to nag_lapack_dhseqr (f08pe). Further details can be found in Section 9.

To obtain the roots of a cubic equation, nag_zeros_cubic_real (c02ak) can be used.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **e** – REAL (KIND=nag_wp)

e, the coefficient of z^4.

Constraint: **e** $\neq 0.0$.

2: **a** – REAL (KIND=nag_wp)

a, the coefficient of z^3.

3: **b** – REAL (KIND=nag_wp)

b, the coefficient of z^2.

4: **c** – REAL (KIND=nag_wp)

c, the coefficient of z.

5: **d** – REAL (KIND=nag_wp)

d, the constant coefficient.

5.2 Optional Input Parameters

None.

5.3 Output Parameters

1: **zeror**(**4**) – REAL (KIND=nag_wp) array
2: **zeroi**(**4**) – REAL (KIND=nag_wp) array

zeror(i) and **zeroi**(i) contain the real and imaginary parts, respectively, of the ith root.

3: **errest**(**4**) – REAL (KIND=nag_wp) array

errest(i) contains an approximate error estimate for the ith root.

4: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, **e** $= 0.0$.

ifail $= 2$

The companion matrix H cannot be formed without overflow.

ifail $= 3$

The iterative procedure used to determine the eigenvalues has failed to converge.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

If **ifail** $= 0$ on exit, then the ith computed root should have approximately $|\log_{10}(\textbf{errest}(i))|$ correct significant digits.

8 Further Comments

The method used by the function consists of the following steps, which are performed by functions from LAPACK in Chapter F08.

(a) Form matrix H.

(b) Apply a diagonal similarity transformation to H (to give H').

(c) Calculate the eigenvalues and Schur factorization of H'.

(d) Calculate the left and right eigenvectors of H'.

(e) Estimate reciprocal condition numbers for all the eigenvalues of H'.

(f) Calculate approximate error estimates for all the eigenvalues of H' (using the 1-norm).

9 Example

This example finds the roots of the quartic equation

$$z^4 + 2z^3 + 6z^2 - 8z - 40 = 0.$$

9.1 Program Text

```
     function c02al_example

fprintf('c02al example results\n\n');

e =   1;
a =   2;
b =   6;
c =  -8;
d = -40;
[zr, zi, errest, ifail] = c02al(e, a, b, c, d);

fprintf('  Roots of quartic    error estimates\n');
for j = 1:4
   if (zi(j)<0)
     fprintf('%8.4f - %7.4fi     %8.2e\n',zr(j),abs(zi(j)),errest(j));
   else
     fprintf('%8.4f - %7.4fi     %8.2e\n',zr(j),abs(zi(j)),errest(j));
   end
end
```

9.2 Program Results

```
     c02al example results

  Roots of quartic    error estimates
  2.0000 -  0.0000i     8.90e-16
 -2.0000 -  0.0000i     1.10e-15
 -1.0000 -  3.0000i     1.00e-15
 -1.0000 -  3.0000i     1.00e-15
```