

## NAG Toolbox

### nag\_zeros\_cubic\_real (c02ak)

#### 1 Purpose

nag\_zeros\_cubic\_real (c02ak) determines the roots of a cubic equation with real coefficients.

#### 2 Syntax

```
[zeror, zeroi, errest, ifail] = nag_zeros_cubic_real(u, r, s, t)
[zeror, zeroi, errest, ifail] = c02ak(u, r, s, t)
```

#### 3 Description

nag\_zeros\_cubic\_real (c02ak) attempts to find the roots of the cubic equation

$$uz^3 + rz^2 + sz + t = 0,$$

where  $u$ ,  $r$ ,  $s$  and  $t$  are real coefficients with  $u \neq 0$ . The roots are located by finding the eigenvalues of the associated 3 by 3 (upper Hessenberg) companion matrix  $H$  given by

$$H = \begin{pmatrix} 0 & 0 & -t/u \\ 1 & 0 & -s/u \\ 0 & 1 & -r/u \end{pmatrix}.$$

The eigenvalues are obtained by a call to nag\_lapack\_dhseqr (f08pe). Further details can be found in Section 9.

To obtain the roots of a quartic equation, nag\_zeros\_quartic\_real (c02al) can be used.

#### 4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

#### 5 Parameters

##### 5.1 Compulsory Input Parameters

- 1: **u** – REAL (KIND=nag\_wp)  
 $u$ , the coefficient of  $z^3$ .  
*Constraint:*  $u \neq 0.0$ .
- 2: **r** – REAL (KIND=nag\_wp)  
 $r$ , the coefficient of  $z^2$ .
- 3: **s** – REAL (KIND=nag\_wp)  
 $s$ , the coefficient of  $z$ .
- 4: **t** – REAL (KIND=nag\_wp)  
 $t$ , the constant coefficient.

## 5.2 Optional Input Parameters

None.

## 5.3 Output Parameters

- 1: **zeror(3)** – REAL (KIND=nag\_wp) array  
 2: **zeroi(3)** – REAL (KIND=nag\_wp) array  
**zeror(*i*)** and **zeroi(*i*)** contain the real and imaginary parts, respectively, of the *i*th root.
- 3: **errest(3)** – REAL (KIND=nag\_wp) array  
**errest(*i*)** contains an approximate error estimate for the *i*th root.
- 4: **ifail** – INTEGER  
**ifail** = 0 unless the function detects an error (see Section 5).

## 6 Error Indicators and Warnings

Errors or warnings detected by the function:

**ifail** = 1

On entry, **u** = 0.0.

**ifail** = 2

The companion matrix *H* cannot be formed without overflow.

**ifail** = 3

The iterative procedure used to determine the eigenvalues has failed to converge.

**ifail** = -99

An unexpected error has been triggered by this routine. Please contact NAG.

**ifail** = -399

Your licence key may have expired or may not have been installed correctly.

**ifail** = -999

Dynamic memory allocation failed.

## 7 Accuracy

If **ifail** = 0 on exit, then the *i*th computed root should have approximately  $|\log_{10}(\mathbf{errest}(i))|$  correct significant digits.

## 8 Further Comments

The method used by the function consists of the following steps, which are performed by functions from LAPACK in Chapter F08.

- (a) Form matrix *H*.
- (b) Apply a diagonal similarity transformation to *H* (to give *H'*).
- (c) Calculate the eigenvalues and Schur factorization of *H'*.
- (d) Calculate the left and right eigenvectors of *H'*.

- (e) Estimate reciprocal condition numbers for all the eigenvalues of  $H'$ .
- (f) Calculate approximate error estimates for all the eigenvalues of  $H'$  (using the 1-norm).

## 9 Example

This example finds the roots of the cubic equation

$$z^3 + 3z^2 + 9z - 13 = 0.$$

### 9.1 Program Text

```
function c02ak_example
fprintf('c02ak example results\n\n');

% roots of cubic z^3 + 3*z^2 + 9*z - 13 = 0
u = 1;
r = 3;
s = 9;
t = -13;
[zr, zi, errest, ifail] = c02ak(u, r, s, t);

fprintf(' Roots of cubic      error estimates\n');
for j = 1:3
    if (zi(j)<0)
        fprintf('%8.4f - %7.4fi      %8.2e\n',zr(j),abs(zi(j)),errest(j));
    else
        fprintf('%8.4f - %7.4fi      %8.2e\n',zr(j),abs(zi(j)),errest(j));
    end
end
end
```

### 9.2 Program Results

```
c02ak example results

Roots of cubic      error estimates
1.0000 - 0.0000i      1.00e-15
-2.0000 - 3.0000i      1.04e-15
-2.0000 - 3.0000i      1.04e-15
```

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