

NAG Library Routine Document

F07FSF (ZPOTRS)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

1 Purpose

F07FSF (ZPOTRS) solves a complex Hermitian positive definite system of linear equations with multiple right-hand sides,

$$AX = B,$$

where A has been factorized by F07FRF (ZPOTRF).

2 Specification

SUBROUTINE F07FSF (UPLO, N, NRHS, A, LDA, B, LDB, INFO)

INTEGER N, NRHS, LDA, LDB, INFO
 COMPLEX (KIND=nag_wp) A(LDA,*), B(LDB,*)
 CHARACTER(1) UPLO

The routine may be called by its LAPACK name *zpotrs*.

3 Description

F07FSF (ZPOTRS) is used to solve a complex Hermitian positive definite system of linear equations $AX = B$, this routine must be preceded by a call to F07FRF (ZPOTRF) which computes the Cholesky factorization of A . The solution X is computed by forward and backward substitution.

If UPLO = 'U', $A = U^H U$, where U is upper triangular; the solution X is computed by solving $U^H Y = B$ and then $UX = Y$.

If UPLO = 'L', $A = LL^H$, where L is lower triangular; the solution X is computed by solving $LY = B$ and then $L^H X = Y$.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

1: UPLO – CHARACTER(1) *Input*

On entry: specifies how A has been factorized.

UPLO = 'U'

$A = U^H U$, where U is upper triangular.

UPLO = 'L'

$A = LL^H$, where L is lower triangular.

Constraint: UPLO = 'U' or 'L'.

- 2: N – INTEGER *Input*
On entry: n , the order of the matrix A .
Constraint: $N \geq 0$.
- 3: NRHS – INTEGER *Input*
On entry: r , the number of right-hand sides.
Constraint: $NRHS \geq 0$.
- 4: A(LDA,*) – COMPLEX (KIND=nag_wp) array *Input*
Note: the second dimension of the array A must be at least $\max(1, N)$.
On entry: the Cholesky factor of A , as returned by F07FRF (ZPOTRF).
- 5: LDA – INTEGER *Input*
On entry: the first dimension of the array A as declared in the (sub)program from which F07FSF (ZPOTRS) is called.
Constraint: $LDA \geq \max(1, N)$.
- 6: B(LDB,*) – COMPLEX (KIND=nag_wp) array *Input/Output*
Note: the second dimension of the array B must be at least $\max(1, NRHS)$.
On entry: the n by r right-hand side matrix B .
On exit: the n by r solution matrix X .
- 7: LDB – INTEGER *Input*
On entry: the first dimension of the array B as declared in the (sub)program from which F07FSF (ZPOTRS) is called.
Constraint: $LDB \geq \max(1, N)$.
- 8: INFO – INTEGER *Output*
On exit: $INFO = 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If $INFO = -i$, the i th parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

For each right-hand side vector b , the computed solution x is the exact solution of a perturbed system of equations $(A + E)x = b$, where

if UPLO = 'U', $|E| \leq c(n)\epsilon|U^H||U|$;

if UPLO = 'L', $|E| \leq c(n)\epsilon|L||L^H|$,

$c(n)$ is a modest linear function of n , and ϵ is the *machine precision*.

If \hat{x} is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x - \hat{x}\|_{\infty}}{\|x\|_{\infty}} \leq c(n) \text{cond}(A, x)\epsilon$$

where $\text{cond}(A, x) = \frac{\|A^{-1}\|_{\infty}\|A\|_{\infty}\|x\|_{\infty}}{\|x\|_{\infty}} \leq \text{cond}(A) = \frac{\|A^{-1}\|_{\infty}\|A\|_{\infty}}{\|x\|_{\infty}} \leq \kappa_{\infty}(A)$.

Note that $\text{cond}(A, x)$ can be much smaller than $\text{cond}(A)$.

Forward and backward error bounds can be computed by calling F07FVF (ZPORFS), and an estimate for $\kappa_{\infty}(A)$ ($= \kappa_1(A)$) can be obtained by calling F07FUF (ZPOCON).

8 Further Comments

The total number of real floating point operations is approximately $8n^2r$.

This routine may be followed by a call to F07FVF (ZPORFS) to refine the solution and return an error estimate.

The real analogue of this routine is F07FEF (DPOTRS).

9 Example

This example solves the system of equations $AX = B$, where

$$A = \begin{pmatrix} 3.23 + 0.00i & 1.51 - 1.92i & 1.90 + 0.84i & 0.42 + 2.50i \\ 1.51 + 1.92i & 3.58 + 0.00i & -0.23 + 1.11i & -1.18 + 1.37i \\ 1.90 - 0.84i & -0.23 - 1.11i & 4.09 + 0.00i & 2.33 - 0.14i \\ 0.42 - 2.50i & -1.18 - 1.37i & 2.33 + 0.14i & 4.29 + 0.00i \end{pmatrix}$$

and

$$B = \begin{pmatrix} 3.93 - 6.14i & 1.48 + 6.58i \\ 6.17 + 9.42i & 4.65 - 4.75i \\ -7.17 - 21.83i & -4.91 + 2.29i \\ 1.99 - 14.38i & 7.64 - 10.79i \end{pmatrix}.$$

Here A is Hermitian positive definite and must first be factorized by F07FRF (ZPOTRF).

9.1 Program Text

```

Program f07fsfe

!       F07FSF Example Program Text

!       Mark 24 Release. NAG Copyright 2012.

!       .. Use Statements ..
!       Use nag_library, Only: nag_wp, x04dbf, zpotrf, zpotrs
!       .. Implicit None Statement ..
!       Implicit None
!       .. Parameters ..
!       Integer, Parameter          :: nin = 5, nout = 6
!       .. Local Scalars ..
!       Integer                    :: i, ifail, info, lda, ldb, n, nrhs
!       Character (1)              :: uplo
!       .. Local Arrays ..
!       Complex (Kind=nag_wp), Allocatable :: a(:,,:), b(:,,:)
!       Character (1)                :: clabs(1), rlabs(1)
!       .. Executable Statements ..
!       Write (nout,*) 'F07FSF Example Program Results'
!       Skip heading in data file
!       Read (nin,*)
!       Read (nin,*) n, nrhs
!       lda = n
!       ldb = n

```

```

        Allocate (a(lda,n),b(ldb,nrhs))

!       Read A and B from data file

        Read (nin,*) uplo
        If (uplo=='U') Then
            Read (nin,*)(a(i,i:n),i=1,n)
        Else If (uplo=='L') Then
            Read (nin,*)(a(i,1:i),i=1,n)
        End If
        Read (nin,*)(b(i,1:nrhs),i=1,n)

!       Factorize A
!       The NAG name equivalent of zpotrf is f07frf
        Call zpotrf(uplo,n,a,lda,info)

        Write (nout,*)
        Flush (nout)
        If (info==0) Then

!           Compute solution
!           The NAG name equivalent of zpotrs is f07fsf
            Call zpotrs(uplo,n,nrhs,a,lda,b,ldb,info)

!           Print solution

!           ifail: behaviour on error exit
!           =0 for hard exit, =1 for quiet-soft, =-1 for noisy-soft
            ifail = 0
            Call x04dbf('General',' ',n,nrhs,b,ldb,'Bracketed','F7.4', &
                'Solution(s)','Integer',rlabs,'Integer',clabs,80,0,ifail)

        Else
            Write (nout,*) 'A is not positive definite'
        End If

    End Program f07fsfe

```

9.2 Program Data

F07FSF Example Program Data

```

    4 2                                     :Values of N and NRHS
    'L'                                     :Value of UPLO
(3.23, 0.00)
(1.51, 1.92) ( 3.58, 0.00)
(1.90,-0.84) (-0.23,-1.11) ( 4.09, 0.00)
(0.42,-2.50) (-1.18,-1.37) ( 2.33, 0.14) ( 4.29, 0.00) :End of matrix A
( 3.93, -6.14) ( 1.48,  6.58)
( 6.17,  9.42) ( 4.65, -4.75)
(-7.17,-21.83) (-4.91,  2.29)
( 1.99,-14.38) ( 7.64,-10.79)                :End of matrix B

```

9.3 Program Results

F07FSF Example Program Results

```

Solution(s)
           1           2
1 ( 1.0000,-1.0000) (-1.0000, 2.0000)
2 (-0.0000, 3.0000) ( 3.0000,-4.0000)
3 (-4.0000,-5.0000) (-2.0000, 3.0000)
4 ( 2.0000, 1.0000) ( 4.0000,-5.0000)

```
