NAG Library Routine Document

F07FDF (DPOTRF)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07FDF (DPOTRF) computes the Cholesky factorization of a real symmetric positive-definite matrix.

2 Specification

```
SUBROUTINE F07FDF(UPLO, N, A, LDA, INFO)
INTEGER           N, LDA, INFO
double precision  A(LDA,*)
CHARACTER*1       UPLO
```

The routine may be called by its LAPACK name ***dpotrf***.

3 Description

F07FDF (DPOTRF) forms the Cholesky factorization of a real symmetric positive-definite matrix A either as $A = U^{\mathrm{T}}U$ if UPLO = 'U' or $A = LL^{\mathrm{T}}$ if UPLO = 'L', where U is an upper triangular matrix and L is lower triangular.

4 References

Demmel J W (1989) On floating-point errors in Cholesky *LAPACK Working Note No. 14* University of Tennessee, Knoxville

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

1: UPLO – CHARACTER*1 *Input*

On entry: indicates whether the upper or lower triangular part of A is stored and how A is to be factorized.

UPLO = 'U'

The upper triangular part of A is stored and A is factorized as $U^{\mathrm{T}}U$, where U is upper triangular.

UPLO = 'L'

The lower triangular part of A is stored and A is factorized as LL^{T}, where L is lower triangular.

Constraint: UPLO = 'U' or 'L'.

2: N – INTEGER *Input*

On entry: n, the order of the matrix A.

Constraint: N $\geq$ 0.

3: A(LDA,$*$) – ***double precision*** array *Input/Output*

Note: the second dimension of the array A must be at least max(1, N).

On entry: the n by n symmetric positive-definite matrix A.

If UPLO = 'U', the upper triangular part of A must be stored and the elements of the array below the diagonal are not referenced.

If UPLO = 'L', the lower triangular part of A must be stored and the elements of the array above the diagonal are not referenced.

On exit: the upper or lower triangle of A is overwritten by the Cholesky factor U or L as specified by UPLO.

4: LDA – INTEGER *Input*

On entry: the first dimension of the array A as declared in the (sub)program from which F07FDF (DPOTRF) is called.

Constraint: LDA $\geq$ max(1, N).

5: INFO – INTEGER *Output*

On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO $= -i$, the ith parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If INFO $= i$, the leading minor of order i is not positive-definite and the factorization could not be completed. Hence A itself is not positive-definite. This may indicate an error in forming the matrix A. To factorize a matrix which is not positive-definite, call F07MDF (DSYTRF) instead.

7 Accuracy

If UPLO = 'U', the computed factor U is the exact factor of a perturbed matrix $A + E$, where

$$|E| \leq c(n)\epsilon \left|U^{\mathrm{T}}\right||U|,$$

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***. If UPLO = 'L', a similar statement holds for the computed factor L. It follows that $\left|e_{ij}\right| \leq c(n)\epsilon\sqrt{a_{ii}a_{jj}}$.

8 Further Comments

The total number of floating-point operations is approximately $\frac{1}{3}n^3$.

A call to F07FDF (DPOTRF) may be followed by calls to the routines:

F07FEF (DPOTRS) to solve $AX = B$;

F07FGF (DPOCON) to estimate the condition number of A;

F07FJF (DPOTRI) to compute the inverse of A.

The complex analogue of this routine is F07FRF (ZPOTRF).

9 Example

This example computes the Cholesky factorization of the matrix A, where

$$A = \begin{pmatrix} 4.16 & -3.12 & 0.56 & -0.10 \\ -3.12 & 5.03 & -0.83 & 1.18 \\ 0.56 & -0.83 & 0.76 & 0.34 \\ -0.10 & 1.18 & 0.34 & 1.18 \end{pmatrix}.$$

9.1 Program Text

```
*     F07FDF Example Program Text
*     Mark 15 Release. NAG Copyright 1991.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          NMAX, LDA
      PARAMETER        (NMAX=8,LDA=NMAX)
*     .. Local Scalars ..
      INTEGER          I, IFAIL, INFO, J, N
      CHARACTER        UPLO
*     .. Local Arrays ..
      DOUBLE PRECISION A(LDA,NMAX)
*     .. External Subroutines ..
      EXTERNAL         DPOTRF, X04CAF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'F07FDF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) N
      IF (N.LE.NMAX) THEN
*
*        Read A from data file
*
         READ (NIN,*) UPLO
         IF (UPLO.EQ.'U') THEN
            READ (NIN,*) ((A(I,J),J=I,N),I=1,N)
         ELSE IF (UPLO.EQ.'L') THEN
            READ (NIN,*) ((A(I,J),J=1,I),I=1,N)
         END IF
*
*        Factorize A
*
         CALL DPOTRF(UPLO,N,A,LDA,INFO)
*
         WRITE (NOUT,*)
         IF (INFO.EQ.0) THEN
*
*           Print factor
*
            IFAIL = 0
            CALL X04CAF(UPLO,'Nonunit',N,N,A,LDA,'Factor',IFAIL)
         ELSE
            WRITE (NOUT,*) 'A is not positive-definite'
         END IF
      END IF
*
      END
```

9.2 Program Data

```
F07FDF Example Program Data
  4                           :Value of N
  'L'                         :Value of UPLO
  4.16
 -3.12   5.03
  0.56  -0.83   0.76
 -0.10   1.18   0.34   1.18   :End of matrix A
```

9.3 Program Results

```
 F07FDF Example Program Results

 Factor
             1          2          3          4
 1      2.0396
 2     -1.5297     1.6401
 3      0.2746    -0.2500     0.7887
 4     -0.0490     0.6737     0.6617     0.5347
```