

# NAG Library Routine Document

## C06RCF

**Note:** before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

### 1 Purpose

C06RCF computes the discrete quarter-wave Fourier sine transforms of  $m$  sequences of real data values.

### 2 Specification

SUBROUTINE C06RCF (DIRECT, M, N, X, WORK, IFAIL)

INTEGER M, N, IFAIL  
 REAL (KIND=nag\_wp) X(M\*(N+2)), WORK(\*)  
 CHARACTER(1) DIRECT

### 3 Description

Given  $m$  sequences of  $n$  real data values  $x_j^p$ , for  $j = 1, 2, \dots, n$  and  $p = 1, 2, \dots, m$ , C06RCF simultaneously calculates the quarter-wave Fourier sine transforms of all the sequences defined by

$$\hat{x}_k^p = \frac{1}{\sqrt{n}} \left( \sum_{j=1}^{n-1} x_j^p \times \sin \left( j(2k-1) \frac{\pi}{2n} \right) + \frac{1}{2} 1^{k-1} x_n^p \right), \quad \text{if DIRECT = 'F'}$$

or its inverse

$$x_k^p = \frac{2}{\sqrt{n}} \sum_{j=1}^n \hat{x}_j^p \times \sin \left( (2j-1) k \frac{\pi}{2n} \right), \quad \text{if DIRECT = 'B'}$$

where  $k = 1, 2, \dots, n$  and  $p = 1, 2, \dots, m$ .

(Note the scale factor  $\frac{1}{\sqrt{n}}$  in this definition.)

A call of C06RCF with DIRECT = 'F' followed by a call with DIRECT = 'B' will restore the original data.

The transform calculated by this routine can be used to solve Poisson's equation when the solution is specified at the left boundary, and the derivative of the solution is specified at the right boundary (see Swarztrauber (1977)).

The routine uses a variant of the fast Fourier transform (FFT) algorithm (see Brigham (1974)) known as the Stockham self-sorting algorithm, described in Temperton (1983), together with pre- and post-processing stages described in Swarztrauber (1982). Special coding is provided for the factors 2, 3, 4 and 5.

### 4 References

Brigham E O (1974) *The Fast Fourier Transform* Prentice-Hall

Swarztrauber P N (1977) The methods of cyclic reduction, Fourier analysis and the FACR algorithm for the discrete solution of Poisson's equation on a rectangle *SIAM Rev.* **19(3)** 490–501

Swarztrauber P N (1982) Vectorizing the FFT's *Parallel Computation* (ed G Rodrigue) 51–83 Academic Press

Temperton C (1983) Fast mixed-radix real Fourier transforms *J. Comput. Phys.* **52** 340–350

## 5 Parameters

- 1: DIRECT – CHARACTER(1) *Input*  
*On entry:* if the forward transform as defined in Section 3 is to be computed, then DIRECT must be set equal to 'F'.  
 If the backward transform is to be computed then DIRECT must be set equal to 'B'.  
*Constraint:* DIRECT = 'F' or 'B'.
- 2: M – INTEGER *Input*  
*On entry:*  $m$ , the number of sequences to be transformed.  
*Constraint:*  $M \geq 1$ .
- 3: N – INTEGER *Input*  
*On entry:*  $n$ , the number of real values in each sequence.  
*Constraint:*  $N \geq 1$ .
- 4: X( $M \times (N + 2)$ ) – REAL (KIND=nag\_wp) array *Input/Output*  
*On entry:* the data must be stored in X as if in a two-dimensional array of dimension (1 : M, 1 : N + 2); each of the  $m$  sequences is stored in a **row** of the array. In other words, if the data values of the  $p$ th sequence to be transformed are denoted by  $x_j^p$ , for  $j = 1, 2, \dots, n$  and  $p = 1, 2, \dots, m$ , then the first  $mn$  elements of the array X must contain the values
- $$x_1^1, x_1^2, \dots, x_1^m, x_2^1, x_2^2, \dots, x_2^m, \dots, x_n^1, x_n^2, \dots, x_n^m.$$
- The  $(n + 1)$ th and  $(n + 2)$ th elements of each row  $x_{n+1}^p, x_{n+2}^p$ , for  $p = 1, 2, \dots, m$ , are required as workspace. These  $2m$  elements may contain arbitrary values as they are set to zero by the routine.  
*On exit:* the  $m$  quarter-wave sine transforms stored as if in a two-dimensional array of dimension (1 : M, 1 : N + 2). Each of the  $m$  transforms is stored in a **row** of the array, overwriting the corresponding original sequence. If the  $n$  components of the  $p$ th quarter-wave sine transform are denoted by  $\hat{x}_k^p$ , for  $k = 1, 2, \dots, n$  and  $p = 1, 2, \dots, m$ , then the  $m(n + 2)$  elements of the array X contain the values
- $$\hat{x}_1^1, \hat{x}_1^2, \dots, \hat{x}_1^m, \hat{x}_2^1, \hat{x}_2^2, \dots, \hat{x}_2^m, \dots, \hat{x}_n^1, \hat{x}_n^2, \dots, \hat{x}_n^m, 0, 0, \dots, 0 \text{ (} 2m \text{ times)}.$$
- 5: WORK(\*) – REAL (KIND=nag\_wp) array *Workspace*  
**Note:** the dimension of the array WORK must be at least  $M \times N + 2 \times N + 15$ .  
 The workspace requirements as documented for C06RCF may be an overestimate in some implementations.  
*On exit:* WORK(1) contains the minimum workspace required for the current values of M and N with this implementation.
- 6: IFAIL – INTEGER *Input/Output*  
*On entry:* IFAIL must be set to 0, -1 or 1. If you are unfamiliar with this parameter you should refer to Section 3.3 in the Essential Introduction for details.  
 For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, if you are not familiar with this parameter, the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**  
*On exit:* IFAIL = 0 unless the routine detects an error or a warning has been flagged (see Section 6).

## 6 Error Indicators and Warnings

If on entry  $IFAIL = 0$  or  $-1$ , explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

$IFAIL = 1$

On entry,  $M < 1$ .

$IFAIL = 2$

On entry,  $N < 1$ .

$IFAIL = 3$

On entry,  $DIRECT \neq 'F'$  or  $'B'$ .

$IFAIL = 4$

An unexpected error has occurred in an internal call. Check all subroutine calls and array dimensions. Seek expert help.

## 7 Accuracy

Some indication of accuracy can be obtained by performing a subsequent inverse transform and comparing the results with the original sequence (in exact arithmetic they would be identical).

## 8 Further Comments

The time taken by C06RCF is approximately proportional to  $nm \log n$ , but also depends on the factors of  $n$ . C06RCF is fastest if the only prime factors of  $n$  are 2, 3 and 5, and is particularly slow if  $n$  is a large prime, or has large prime factors.

## 9 Example

This example reads in sequences of real data values and prints their quarter-wave sine transforms as computed by C06RCF with  $DIRECT = 'F'$ . It then calls the routine again with  $DIRECT = 'B'$  and prints the results which may be compared with the original data.

### 9.1 Program Text

```

Program c06rcfe

!      C06RCF Example Program Text

!      Mark 24 Release. NAG Copyright 2012.

!      .. Use Statements ..
      Use nag_library, Only: c06rcf, nag_wp
!      .. Implicit None Statement ..
      Implicit None
!      .. Parameters ..
      Integer, Parameter          :: nin = 5, nout = 6
!      .. Local Scalars ..
      Integer                      :: i, ieof, ifail, j, m, n
!      .. Local Arrays ..
      Real (Kind=nag_wp), Allocatable :: work(:), x(:)
!      .. Executable Statements ..
      Write (nout,*) 'C06RCF Example Program Results'
!      Skip heading in data file
      Read (nin,*)
loop: Do

```

```

Read (nin,*,Iostat=ieof) m, n
If (ieof<0) Exit loop

Allocate (x(m*(n+2)),work(m*n+2*n+15))
Do j = 1, m
  Read (nin,*)(x(i*m+j),i=0,n-1)
End Do
Write (nout,*)
Write (nout,*) 'Original data values'
Write (nout,*)
Do j = 1, m
  Write (nout,99999)(x(i*m+j),i=0,n-1)
End Do

!      ifail: behaviour on error exit
!      =0 for hard exit, =1 for quiet-soft, =-1 for noisy-soft
ifail = 0
!      -- Compute transform
Call c06rcf('Forward',m,n,x,work,ifail)

Write (nout,*)
Write (nout,*) 'Discrete quarter-wave Fourier sine transforms'
Write (nout,*)
Do j = 1, m
  Write (nout,99999)(x(i*m+j),i=0,n-1)
End Do

!      -- Compute inverse transform
Call c06rcf('Backward',m,n,x,work,ifail)

Write (nout,*)
Write (nout,*) 'Original data as restored by inverse transform'
Write (nout,*)
Do j = 1, m
  Write (nout,99999)(x(i*m+j),i=0,n-1)
End Do
Deallocate (x,work)
End Do loop

99999 Format (6X,7F10.4)
End Program c06rcfe

```

## 9.2 Program Data

```

C06RCF Example Program Data
 3  6                               : m, n
0.3854 0.6772 0.1138 0.6751 0.6362 0.1424
0.5417 0.2983 0.1181 0.7255 0.8638 0.8723
0.9172 0.0644 0.6037 0.6430 0.0428 0.4815 : x

```

## 9.3 Program Results

C06RCF Example Program Results

Original data values

0.3854	0.6772	0.1138	0.6751	0.6362	0.1424
0.5417	0.2983	0.1181	0.7255	0.8638	0.8723
0.9172	0.0644	0.6037	0.6430	0.0428	0.4815

Discrete quarter-wave Fourier sine transforms

0.7304	0.2078	0.1150	0.2577	-0.2869	-0.0815
0.9274	-0.1152	0.2532	0.2883	-0.0026	-0.0635
0.6268	0.3547	0.0760	0.3078	0.4987	-0.0507

Original data as restored by inverse transform

0.3854	0.6772	0.1138	0.6751	0.6362	0.1424
0.5417	0.2983	0.1181	0.7255	0.8638	0.8723
0.9172	0.0644	0.6037	0.6430	0.0428	0.4815

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