

# NAG Library Function Document

## nag\_mesh2d\_renum (d06ccc)

### 1 Purpose

nag\_mesh2d\_renum (d06ccc) renumbers the vertices of a given mesh using a Gibbs method, in order to reduce the bandwidth of Finite Element matrices associated with that mesh.

### 2 Specification

```
#include <nag.h>
#include <nagd06.h>

void nag_mesh2d_renum (Integer nv, Integer nelt, Integer nedge,
                      Integer nnzmax, Integer *nnz, double coor[], Integer edge[],
                      Integer conn[], Integer irow[], Integer icol[], Integer itrace,
                      const char *outfile, NagError *fail)
```

### 3 Description

nag\_mesh2d\_renum (d06ccc) uses a Gibbs method to renumber the vertices of a given mesh in order to reduce the bandwidth of the associated finite element matrix  $A$ . This matrix has elements  $a_{ij}$  such that:

$$a_{ij} \neq 0 \Rightarrow i \text{ and } j \text{ are vertices belonging to the same triangle.}$$

This function reduces the bandwidth  $m$ , which is the smallest integer such that  $a_{ij} \neq 0$  whenever  $|i - j| > m$  (see Gibbs *et al.* (1976) for details about that method). nag\_mesh2d\_renum (d06ccc) also returns the sparsity structure of the matrix associated with the renumbered mesh.

This function is derived from material in the MODULEF package from INRIA (Institut National de Recherche en Informatique et Automatique).

### 4 References

Gibbs N E, Poole W G Jr and Stockmeyer P K (1976) An algorithm for reducing the bandwidth and profile of a sparse matrix *SIAM J. Numer. Anal.* **13** 236–250

### 5 Arguments

- |    |   |              |
|----|---|--------------|
| 1: | <b>nv</b> – Integer   | <i>Input</i> |
|    | <i>On entry:</i> the total number of vertices in the input mesh.  |              |
|    | <i>Constraint:</i> <b>nv</b> $\geq$ 3.                            |              |
| 2: | <b>nelt</b> – Integer   | <i>Input</i> |
|    | <i>On entry:</i> the number of triangles in the input mesh.       |              |
|    | <i>Constraint:</i> <b>nelt</b> $\leq$ $2 \times$ <b>nv</b> $-$ 1. |              |
| 3: | <b>nedge</b> – Integer  | <i>Input</i> |
|    | <i>On entry:</i> the number of boundary edges in the input mesh.  |              |
|    | <i>Constraint:</i> <b>nedge</b> $\geq$ 1.                         |              |

- 4: **nnzmax** – Integer *Input*  
*On entry:* the maximum number of nonzero entries in the matrix based on the input mesh. It is the dimension of the arrays **irow** and **icol** as declared in the function from which `nag_mesh2d_renum` (d06ccc) is called.  
*Constraint:*  $4 \times \mathbf{nelt} + \mathbf{nv} \leq \mathbf{nnzmax} \leq \mathbf{nv}^2$ .
- 5: **nnz** – Integer \* *Output*  
*On exit:* the number of nonzero entries in the matrix based on the input mesh.
- 6: **coor**[ $2 \times \mathbf{nv}$ ] – double *Input/Output*  
**Note:** the  $(i, j)$ th element of the matrix is stored in **coor**[( $j - 1$ )  $\times$  2 +  $i - 1$ ].  
*On entry:* **coor**[( $i - 1$ )  $\times$  2] contains the  $x$  coordinate of the  $i$ th input mesh vertex, for  $i = 1, 2, \dots, \mathbf{nv}$ ; while **coor**[( $i - 1$ )  $\times$  2 + 1] contains the corresponding  $y$  coordinate.  
*On exit:* **coor**[( $i - 1$ )  $\times$  2] will contain the  $x$  coordinate of the  $i$ th renumbered mesh vertex, for  $i = 1, 2, \dots, \mathbf{nv}$ ; while **coor**[( $i - 1$ )  $\times$  2 + 1] will contain the corresponding  $y$  coordinate.
- 7: **edge**[ $3 \times \mathbf{nedge}$ ] – Integer *Input/Output*  
**Note:** the  $(i, j)$ th element of the matrix is stored in **edge**[( $j - 1$ )  $\times$  3 +  $i - 1$ ].  
*On entry:* the specification of the boundary or interface edges. **edge**[( $j - 1$ )  $\times$  3] and **edge**[( $j - 1$ )  $\times$  3 + 1] contain the vertex numbers of the two end points of the  $j$ th boundary edge. **edge**[( $j - 1$ )  $\times$  3 + 2] is a user-supplied tag for the  $j$ th boundary or interface edge: **edge**[( $j - 1$ )  $\times$  3 + 2] = 0 for an interior edge and has a nonzero tag otherwise. Note that the edge vertices are numbered from 1 to **nv**.  
*Constraint:*  $1 \leq \mathbf{edge}[(j - 1) \times 3 + i - 1] \leq \mathbf{nv}$  and  $\mathbf{edge}[(j - 1) \times 3] \neq \mathbf{edge}[(j - 1) \times 3 + 1]$ , for  $i = 1, 2$  and  $j = 1, 2, \dots, \mathbf{nedge}$ .  
*On exit:* the renumbered specification of the boundary or interface edges.
- 8: **conn**[ $3 \times \mathbf{nelt}$ ] – Integer *Input/Output*  
**Note:** the  $(i, j)$ th element of the matrix is stored in **conn**[( $j - 1$ )  $\times$  3 +  $i - 1$ ].  
*On entry:* the connectivity of the mesh between triangles and vertices. For each triangle  $j$ , **conn**[( $j - 1$ )  $\times$  3 +  $i - 1$ ] gives the indices of its three vertices (in anticlockwise order), for  $i = 1, 2, 3$  and  $j = 1, 2, \dots, \mathbf{nelt}$ . Note that the mesh vertices are numbered from 1 to **nv**.  
*Constraint:*  $1 \leq \mathbf{conn}[(j - 1) \times 3 + i - 1] \leq \mathbf{nv}$  and  $\mathbf{conn}[(j - 1) \times 3] \neq \mathbf{conn}[(j - 1) \times 3 + 1]$  and  $\mathbf{conn}[(j - 1) \times 3] \neq \mathbf{conn}[(j - 1) \times 3 + 2]$  and  $\mathbf{conn}[(j - 1) \times 3 + 1] \neq \mathbf{conn}[(j - 1) \times 3 + 2]$ , for  $i = 1, 2, 3$  and  $j = 1, 2, \dots, \mathbf{nelt}$ .  
*On exit:* the renumbered connectivity of the mesh between triangles and vertices.
- 9: **irow**[**nnzmax**] – Integer *Output*  
10: **icol**[**nnzmax**] – Integer *Output*  
*On exit:* the first **nnz** elements contain the row and column indices of the nonzero elements supplied in the finite element matrix  $A$ .
- 11: **itrace** – Integer *Input*  
*On entry:* the level of trace information required from `nag_mesh2d_renum` (d06ccc).  
**itrace**  $\leq 0$   
No output is generated.

**itrace** = 1

Information about the effect of the renumbering on the finite element matrix are output. This information includes the half bandwidth and the sparsity structure of this matrix before and after renumbering.

**itrace** > 1

The output is similar to that produced when **itrace** = 1 but the sparsities (for each row of the matrix, indices of nonzero entries) of the matrix before and after renumbering are also output.

12: **outfile** – const char \* *Input*

*On entry:* the name of a file to which diagnostic output will be directed. If **outfile** is **NULL** the diagnostic output will be directed to standard output.

13: **fail** – NagError \* *Input/Output*

The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

See Section 3.2.1.2 in How to Use the NAG Library and its Documentation for further information.

### NE\_BAD\_PARAM

On entry, argument  $\langle value \rangle$  had an illegal value.

### NE\_FAIL\_SPARSITY

An error has occurred during the computation of the compact sparsity of the finite element matrix. Check the Triangle/Vertices connectivity.

### NE\_INT

On entry, **nedge** =  $\langle value \rangle$ .

Constraint: **nedge**  $\geq 1$ .

On entry, **nv** =  $\langle value \rangle$ .

Constraint: **nv**  $\geq 3$ .

### NE\_INT\_2

On entry, **nelt** =  $\langle value \rangle$  and **nv** =  $\langle value \rangle$ .

Constraint: **nelt**  $\leq 2 \times \mathbf{nv} - 1$ .

On entry, the end points of the edge  $J$  have the same index  $I$ :  $J = \langle value \rangle$  and  $I = \langle value \rangle$ .

On entry, vertices 1 and 2 of the triangle  $K$  have the same index  $I$ :  $K = \langle value \rangle$  and  $I = \langle value \rangle$ .

On entry, vertices 1 and 3 of the triangle  $K$  have the same index  $I$ :  $K = \langle value \rangle$  and  $I = \langle value \rangle$ .

On entry, vertices 2 and 3 of the triangle  $K$  have the same index  $I$ :  $K = \langle value \rangle$  and  $I = \langle value \rangle$ .

### NE\_INT\_3

On entry, **nnzmax** =  $\langle value \rangle$ , **nelt** =  $\langle value \rangle$  and **nv** =  $\langle value \rangle$ .

Constraint: **nnzmax**  $\geq (4 \times \mathbf{nelt} + \mathbf{nv})$  and **nnzmax**  $\leq \mathbf{nv}^2$ .

**NE\_INT\_4**

On entry, **CONN**( $I, J$ ) =  $\langle value \rangle$ ,  $I = \langle value \rangle$ ,  $J = \langle value \rangle$  and **nv** =  $\langle value \rangle$ .

Constraint: **CONN**( $I, J$ )  $\geq 1$  and **CONN**( $I, J$ )  $\leq$  **nv**, where **CONN**( $I, J$ ) denotes **conn**[( $J - 1$ )  $\times$  3 +  $I - 1$ ].

On entry, **EDGE**( $I, J$ ) =  $\langle value \rangle$ ,  $I = \langle value \rangle$ ,  $J = \langle value \rangle$  and **nv** =  $\langle value \rangle$ .

Constraint: **EDGE**( $I, J$ )  $\geq 1$  and **EDGE**( $I, J$ )  $\leq$  **nv**, where **EDGE**( $I, J$ ) denotes **edge**[( $J - 1$ )  $\times$  3 +  $I - 1$ ].

**NE\_INTERNAL\_ERROR**

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.

See Section 3.6.6 in How to Use the NAG Library and its Documentation for further information.

A serious error has occurred in an internal call to the renumbering function. Check the input mesh especially the connectivity. Seek expert help.

**NE\_NO\_LICENCE**

Your licence key may have expired or may not have been installed correctly.

See Section 3.6.5 in How to Use the NAG Library and its Documentation for further information.

**NE\_NOT\_CLOSE\_FILE**

Cannot close file  $\langle value \rangle$ .

**NE\_NOT\_WRITE\_FILE**

Cannot open file  $\langle value \rangle$  for writing.

**7 Accuracy**

Not applicable.

**8 Parallelism and Performance**

nag\_mesh2d\_renum (d06ccc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

Please consult the x06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

**9 Further Comments**

None.

**10 Example**

In this example, a geometry with two holes (two interior circles inside an exterior one) is considered. The geometry has been meshed using the simple incremental method (nag\_mesh2d\_inc (d06aac)) and it has 250 vertices and 402 triangles (see Figure 1 in Section 10.3). The function nag\_mesh2d\_bound (d06bac) is used to renumber the vertices, and one can see the benefit in terms of the sparsity of the finite element matrix based on the renumbered mesh (see Figure 2 and 3 in Section 10.3).

## 10.1 Program Text

```

/* nag_mesh2d_renum (d06ccc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagd06.h>

#define EDGE(I, J) edge[3*((J) -1)+(I) -1]
#define CONN(I, J) conn[3*((J) -1)+(I) -1]
#define COOR(I, J) coor[2*((J) -1)+(I) -1]

int main(void)
{
    Integer exit_status, i, itrace, nedge, nelt, nnz, nnzmax, nv, refTk;
    NagError fail;
    char pmesh[2];
    double *coor = 0;
    Integer *conn = 0, *edge = 0, *icol = 0, *irow = 0;

    INIT_FAIL(fail);

    exit_status = 0;

    printf(" nag_mesh2d_renum (d06ccc) Example Program Results\n\n");
    fflush(stdout);

    /* Skip heading in data file */

#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif

    /* Reading of the geometry */

#ifdef _WIN32
    scanf_s("%" NAG_IFMT "", &nv);
#else
    scanf("%" NAG_IFMT "", &nv);
#endif
#ifdef _WIN32
    scanf_s("%" NAG_IFMT "", &nelt);
#else
    scanf("%" NAG_IFMT "", &nelt);
#endif
#ifdef _WIN32
    scanf_s("%" NAG_IFMT "", &nedge);
#else
    scanf("%" NAG_IFMT "", &nedge);
#endif
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif

    nnzmax = 10 * nv;

    /* Allocate memory */

    if (!(coor = NAG_ALLOC(2 * nv, double)) ||

```

```

        !(conn = NAG_ALLOC(3 * nelt, Integer)) ||
        !(edge = NAG_ALLOC(3 * nedge, Integer)) ||
        !(irow = NAG_ALLOC(nnzmax, Integer)) ||
        !(icol = NAG_ALLOC(nnzmax, Integer))
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }

    for (i = 1; i <= nv; ++i) {
#ifdef _WIN32
        scanf_s("%lf", &COOR(1, i));
#else
        scanf("%lf", &COOR(1, i));
#endif
#ifdef _WIN32
        scanf_s("%lf", &COOR(2, i));
#else
        scanf("%lf", &COOR(2, i));
#endif
#ifdef _WIN32
        scanf_s("%*[\n] ");
#else
        scanf("%*[\n] ");
#endif
    }

    for (i = 1; i <= nelt; ++i) {
#ifdef _WIN32
        scanf_s("%" NAG_IFMT "", &CONN(1, i));
#else
        scanf("%" NAG_IFMT "", &CONN(1, i));
#endif
#ifdef _WIN32
        scanf_s("%" NAG_IFMT "", &CONN(2, i));
#else
        scanf("%" NAG_IFMT "", &CONN(2, i));
#endif
#ifdef _WIN32
        scanf_s("%" NAG_IFMT "", &CONN(3, i));
#else
        scanf("%" NAG_IFMT "", &CONN(3, i));
#endif
#ifdef _WIN32
        scanf_s("%" NAG_IFMT "", &reftk);
#else
        scanf("%" NAG_IFMT "", &reftk);
#endif
#ifdef _WIN32
        scanf_s("%*[\n] ");
#else
        scanf("%*[\n] ");
#endif
    }

    for (i = 1; i <= nedge; ++i) {
#ifdef _WIN32
        scanf_s("%" NAG_IFMT "", &reftk);
#else
        scanf("%" NAG_IFMT "", &reftk);
#endif
#ifdef _WIN32
        scanf_s("%" NAG_IFMT "", &EDGE(1, i));
#else
        scanf("%" NAG_IFMT "", &EDGE(1, i));
#endif
#ifdef _WIN32
        scanf_s("%" NAG_IFMT "", &EDGE(2, i));
#else
        scanf("%" NAG_IFMT "", &EDGE(2, i));

```

```

#endif
#ifdef _WIN32
    scanf_s("%" NAG_IFMT "", &EDGE(3, i));
#else
    scanf("%" NAG_IFMT "", &EDGE(3, i));
#endif
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif
}

#ifdef _WIN32
    scanf_s(" ' %1s '", pmesh, (unsigned)_countof(pmesh));
#else
    scanf(" ' %1s '", pmesh);
#endif
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif

/* Compute the sparsity of the FE matrix */
/* from the input geometry */

/* nag_mesh2d_sparse (d06cbc).
 * Generates a sparsity pattern of a Finite Element matrix
 * associated with a given mesh
 */
nag_mesh2d_sparse(nv, nelt, nnzmax, conn, &nnz, irow, icol, &fail);

if (fail.code == NE_NOERROR) {
    if (pmesh[0] == 'N') {
        printf(" The Matrix Sparsity characteristics\n");
        printf(" before the renumbering\n");
        printf(" nv  =%6" NAG_IFMT "\n", nv);
        printf(" nnz =%6" NAG_IFMT "\n", nnz);
    }
    else if (pmesh[0] == 'Y') {
        /* Output the sparsity of the mesh to view */
        /* it using the NAG Graphics Library */

        printf(" %10" NAG_IFMT "%10" NAG_IFMT "\n", nv, nnz);

        for (i = 0; i < nnz; ++i)
            printf(" %10" NAG_IFMT "%10" NAG_IFMT "\n", irow[i], icol[i]);
    }
    else {
        printf("Problem with the printing option Y or N\n");
        exit_status = -1;
        goto END;
    }
}
else {
    printf("Error from nag_mesh2d_sparse (d06cbc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

/* Call the renumbering routine and get the new sparsity */

itrace = 1;

/* nag_mesh2d_renum (d06ccc).
 * Renumbers a given mesh using Gibbs method
 */
fflush(stdout);
nag_mesh2d_renum(nv, nelt, nedge, nnzmax, &nnz, coor, edge, conn, irow,
                icol, itrace, 0, &fail);

```

```

if (fail.code == NE_NOERROR) {
  if (pmesh[0] == 'N') {
    printf("\n The Matrix Sparsity characteristics\n");
    printf(" after the renumbering\n");
    printf(" nv   =%6" NAG_IFMT "\n", nv);
    printf(" nnz  =%6" NAG_IFMT "\n", nnz);
    printf(" nelt =%6" NAG_IFMT "\n", nelt);
  }
  else if (pmesh[0] == 'Y') {
    /* Output the sparsity of the renumbered mesh */
    /* to view it using the NAG Graphics Library */

    printf("%10" NAG_IFMT "%10" NAG_IFMT "\n", nv, nnz);

    for (i = 0; i < nnz; ++i)
      printf(" %10" NAG_IFMT "%10" NAG_IFMT "\n", irow[i], icol[i]);

    /* Output the renumbered mesh to view */
    /* it using the NAG Graphics Library */

    printf(" %10" NAG_IFMT "%10" NAG_IFMT "\n", nv, nelt);

    for (i = 1; i <= nv; ++i)
      printf("  %15.6e %15.6e  \n", COOR(1, i), COOR(2, i));

    refTk = 0;
    for (i = 1; i <= nelt; ++i)
      printf(" %10" NAG_IFMT "%10" NAG_IFMT "%10" NAG_IFMT "%10" NAG_IFMT
            "\n", CONN(1, i), CONN(2, i), CONN(3, i), refTk);
  }
}
else {
  printf("Error from nag_mesh2d_renum (d06ccc).\n%s\n", fail.message);
  exit_status = 1;
  goto END;
}

END:
  NAG_FREE(coor);
  NAG_FREE(conn);
  NAG_FREE(edge);
  NAG_FREE(irow);
  NAG_FREE(icol);

  return exit_status;
}

```

## 10.2 Program Data

```

D06CCF Example Program Data
      250      402      100      :NV NELT NEDGE
0.100000E+01 0.000000E+00
      .
      .
0.112781E+00 0.103479E+00      :COOR(1:2,1:NV)
      21      55      56      1
      .
      .
      151      250      155      1      :(CONN(:,K), REFT, K=1,...,NELT)
1 1 2 1
      .
      .
100 100 71 1      :(I1, EDGE(:,I), I=1,NEDGE)
'N'      :Printing option 'Y' or 'N'

```



### 10.3 Program Results

nag\_mesh2d\_renum (d06ccc) Example Program Results

The Matrix Sparsity characteristics  
before the renumbering  
nv = 250  
nnz = 1556

Initial half-bandwidth: 234      Initial profile: 18233  
Final half-bandwidth: 28        Final profile: 4038

The Matrix Sparsity characteristics  
after the renumbering  
nv = 250  
nnz = 1556  
nelt = 402

**Example Program**  
Figure 1: Mesh of the Geometry

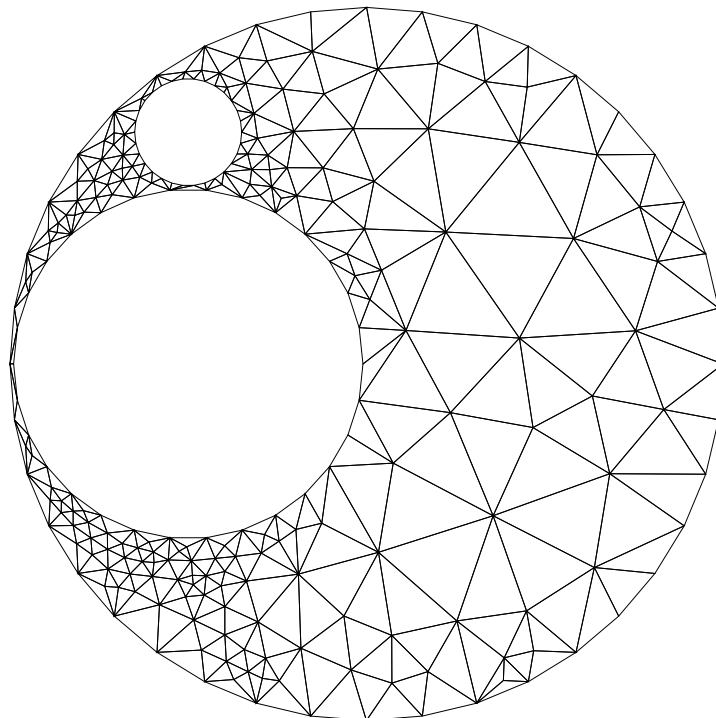


Figure 2: Sparsity of the FE Matrix Before Renumbering

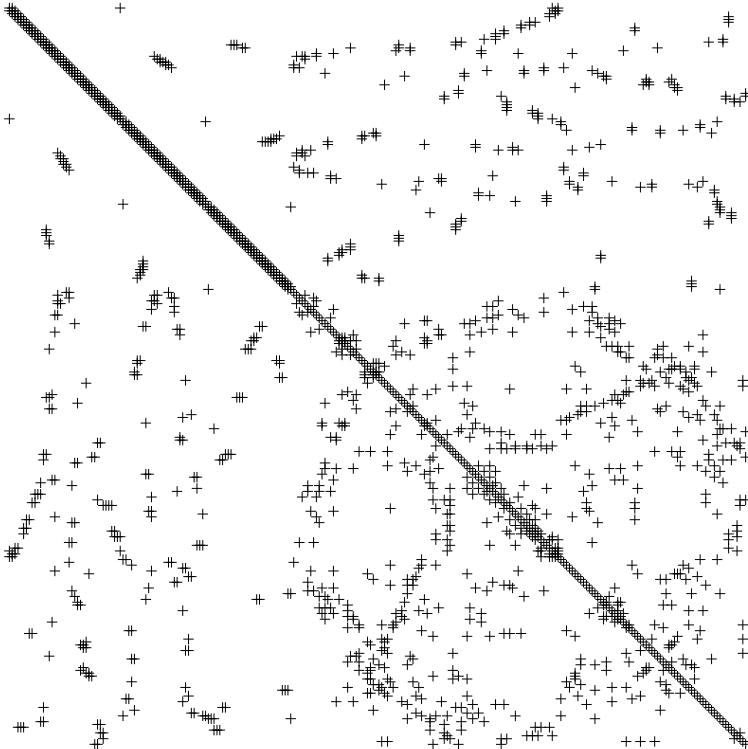


Figure 3: Sparsity of the FE Matrix After Renumbering

