NAG Library Function Document

nag_bsm_greeks (s30abc)

1 Purpose

nag_bsm_greeks (s30abc) computes the European option price given by the Black–Scholes–Merton formula together with its sensitivities (Greeks).

2 Specification

```c
#include <nag.h>
#include <nags.h>

void nag_bsm_greeks (Nag_OrderType order, Nag_CallPut option, Integer m,
Integer n, const double x[], double s, const double t[], double sigma,
double r, double q, double p[], double delta[], double gamma[],
double vega[], double theta[], double rho[], double crho[],
double vanna[], double charm[], double speed[], double colour[],
double zomma[], double vomma[], NagError *fail)
```

3 Description

nag_bsm_greeks (s30abc) computes the price of a European call (or put) option together with the Greeks or sensitivities, which are the partial derivatives of the option price with respect to certain of the other input parameters, by the Black–Scholes–Merton formula (see Black and Scholes (1973) and Merton (1973)). The annual volatility, \( \sigma \), risk-free interest rate, \( r \), and dividend yield, \( q \), must be supplied as input. For a given strike price, \( X \), the price of a European call with underlying price, \( S \), and time to expiry, \( T \), is

\[
P_{\text{call}} = S e^{-qT} \Phi(d_1) - X e^{-rT} \Phi(d_2)
\]

and the corresponding European put price is

\[
P_{\text{put}} = X e^{-rT} \Phi(-d_2) - S e^{-qT} \Phi(-d_1)
\]

and where \( \Phi \) denotes the cumulative Normal distribution function,

\[
\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp(-y^2/2) dy
\]

and

\[
d_1 = \frac{\ln(S/X) + (r-q+\sigma^2/2)T}{\sigma \sqrt{T}},
\]

\[
d_2 = d_1 - \sigma \sqrt{T}.
\]

The option price \( P_{ij} = P(X = X_i, T = T_j) \) is computed for each strike price in a set \( X_i \), \( i = 1, 2, \ldots, m \), and for each expiry time in a set \( T_j \), \( j = 1, 2, \ldots, n \).

4 References


5 Arguments

1: order – Nag_OrderType  
   Input  
   On entry: the order argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by order = Nag_RowMajor. See Section 3.2.1.3 in the Essential Introduction for a more detailed explanation of the use of this argument.
   Constraint: order = Nag_RowMajor or Nag_ColMajor.

2: option – Nag_CallPut  
   Input  
   On entry: determines whether the option is a call or a put.
   option = Nag_Call  
   A call; the holder has a right to buy.
   option = Nag_Put  
   A put; the holder has a right to sell.
   Constraint: option = Nag_Call or Nag_Put.

3: m – Integer  
   Input  
   On entry: the number of strike prices to be used.
   Constraint: m ≥ 1.

4: n – Integer  
   Input  
   On entry: the number of times to expiry to be used.
   Constraint: n ≥ 1.

5: x[m] – const double  
   Input  
   On entry: x[i - 1] must contain X_i, the i\text{th} strike price, for i = 1, 2, \ldots, m.
   Constraint: x[i - 1] ≥ z and x[i - 1] ≤ 1/z, where z = nag_real_safe_small_number, the safe range parameter, for i = 1, 2, \ldots, m.

6: s – double  
   Input  
   On entry: S, the price of the underlying asset.
   Constraint: s ≥ z and s ≤ 1.0/z, where z = nag_real_safe_small_number, the safe range parameter.

7: t[n] – const double  
   Input  
   On entry: t[i - 1] must contain T_i, the i\text{th} time, in years, to expiry, for i = 1, 2, \ldots, n.
   Constraint: t[i - 1] ≥ z, where z = nag_real_safe_small_number, the safe range parameter, for i = 1, 2, \ldots, n.

8: sigma – double  
   Input  
   On entry: \sigma, the volatility of the underlying asset. Note that a rate of 15% should be entered as 0.15.
   Constraint: sigma > 0.0.

9: r – double  
   Input  
   On entry: r, the annual risk-free interest rate, continuously compounded. Note that a rate of 5% should be entered as 0.05.
   Constraint: r ≥ 0.0.
On entry: \( q \), the annual continuous yield rate. Note that a rate of 8% should be entered as 0.08.

Constraint: \( q \geq 0.0 \).

\[ pt, \theta, \gamma, \delta, \rho \]

Note: where \( P(i, j) \) appears in this document, it refers to the array element.

\( [j - 1] \times m + i - 1 \) when \( \text{order} = \text{Nag}_\text{ColMajor} \);
\( [i - 1] \times n + j - 1 \) when \( \text{order} = \text{Nag}_\text{RowMajor} \).

On exit: \( P(i, j) \) contains \( P_{ij} \), the option price evaluated for the strike price \( x_i \) at expiry \( t_j \) for
\( i = 1, 2, \ldots, m \) and \( j = 1, 2, \ldots, n \).

\[ \Delta[m \times n] \]

Note: the \((i, j)\)th element of the matrix is stored in

\( [j - 1] \times m + i - 1 \) when \( \text{order} = \text{Nag}_\text{ColMajor} \);
\( [i - 1] \times n + j - 1 \) when \( \text{order} = \text{Nag}_\text{RowMajor} \).

On exit: the \( m \times n \) array \( \Delta \) contains the sensitivity, \( \frac{\partial P}{\partial \Delta} \), of the option price to change in the
price of the underlying asset.

\[ \Gamma[m \times n] \]

Note: where \( VEGA(i, j) \) appears in this document, it refers to the array element.

\( [j - 1] \times m + i - 1 \) when \( \text{order} = \text{Nag}_\text{ColMajor} \);
\( [i - 1] \times n + j - 1 \) when \( \text{order} = \text{Nag}_\text{RowMajor} \).

On exit: \( VEGA(i, j) \), contains the first-order Greek measuring the sensitivity of the option price
\( P_{ij} \) to change in the volatility of the underlying asset, i.e., \( \frac{\partial P}{\partial \Gamma} \), for \( i = 1, 2, \ldots, m \) and
\( j = 1, 2, \ldots, n \).

\[ \Theta[m \times n] \]

Note: where \( THETA(i, j) \) appears in this document, it refers to the array element.

\( [j - 1] \times m + i - 1 \) when \( \text{order} = \text{Nag}_\text{ColMajor} \);
\( [i - 1] \times n + j - 1 \) when \( \text{order} = \text{Nag}_\text{RowMajor} \).

On exit: \( THETA(i, j) \), contains the first-order Greek measuring the sensitivity of the option price
\( P_{ij} \) to change in time, i.e., \( -\frac{\partial P}{\partial \theta} \), for \( i = 1, 2, \ldots, m \) and \( j = 1, 2, \ldots, n \), where \( b = r - q \).

\[ \Phi[m \times n] \]

Note: where \( RHO(i, j) \) appears in this document, it refers to the array element.

\( [j - 1] \times m + i - 1 \) when \( \text{order} = \text{Nag}_\text{ColMajor} \);
\( [i - 1] \times n + j - 1 \) when \( \text{order} = \text{Nag}_\text{RowMajor} \).

On exit: \( RHO(i, j) \), contains the first-order Greek measuring the sensitivity of the option price \( P_{ij} \)
to change in the annual risk-free interest rate, i.e., \( -\frac{\partial P}{\partial \rho} \), for \( i = 1, 2, \ldots, m \) and \( j = 1, 2, \ldots, n \).
17: \( \text{crho}[m \times n] \) – double

Output

Note: where \( \text{CRHO}(i, j) \) appears in this document, it refers to the array element

\[
\text{crho}(j - 1) \times m + i - 1 \quad \text{when order} = \text{Nag ColMajor};
\]

\[
\text{crho}(i - 1) \times n + j - 1 \quad \text{when order} = \text{Nag RowMajor}.
\]

On exit: \( \text{CRHO}(i, j) \), contains the first-order Greek measuring the sensitivity of the option price \( P_j \) to change in the annual cost of carry rate, i.e., \( -\frac{\partial P_j}{\partial b} \), for \( i = 1, 2, \ldots, m \) and \( j = 1, 2, \ldots, n \), where \( b = r - q \).

18: \( \text{vanna}[m \times n] \) – double

Output

Note: where \( \text{VANNA}(i, j) \) appears in this document, it refers to the array element

\[
\text{vanna}(j - 1) \times m + i - 1 \quad \text{when order} = \text{Nag ColMajor};
\]

\[
\text{vanna}(i - 1) \times n + j - 1 \quad \text{when order} = \text{Nag RowMajor}.
\]

On exit: \( \text{VANNA}(i, j) \), contains the second-order Greek measuring the sensitivity of the first-order Greek \( \Delta_{ij} \) to change in the volatility of the asset price, i.e., \( -\frac{\partial \Delta_{ij}}{\partial \sigma} = -\frac{\partial^2 p_i}{\partial \sigma^2} \), for \( i = 1, 2, \ldots, m \) and \( j = 1, 2, \ldots, n \).

19: \( \text{charm}[m \times n] \) – double

Output

Note: where \( \text{CHARM}(i, j) \) appears in this document, it refers to the array element

\[
\text{charm}(j - 1) \times m + i - 1 \quad \text{when order} = \text{Nag ColMajor};
\]

\[
\text{charm}(i - 1) \times n + j - 1 \quad \text{when order} = \text{Nag RowMajor}.
\]

On exit: \( \text{CHARM}(i, j) \), contains the second-order Greek measuring the sensitivity of the first-order Greek \( \Delta_{ij} \) to change in the time, i.e., \( -\frac{\partial \Delta_{ij}}{\partial \tau} = -\frac{\partial^2 p_i}{\partial \tau^2} \), for \( i = 1, 2, \ldots, m \) and \( j = 1, 2, \ldots, n \).

20: \( \text{speed}[m \times n] \) – double

Output

Note: where \( \text{SPEED}(i, j) \) appears in this document, it refers to the array element

\[
\text{speed}(j - 1) \times m + i - 1 \quad \text{when order} = \text{Nag ColMajor};
\]

\[
\text{speed}(i - 1) \times n + j - 1 \quad \text{when order} = \text{Nag RowMajor}.
\]

On exit: \( \text{SPEED}(i, j) \), contains the third-order Greek measuring the sensitivity of the second-order Greek \( \Gamma_{ij} \) to change in the price of the underlying asset, i.e., \( -\frac{\partial \Gamma_{ij}}{\partial S} = -\frac{\partial^3 p_i}{\partial S^2 \partial \sigma} \), for \( i = 1, 2, \ldots, m \) and \( j = 1, 2, \ldots, n \).

21: \( \text{colour}[m \times n] \) – double

Output

Note: where \( \text{COLOUR}(i, j) \) appears in this document, it refers to the array element

\[
\text{colour}(j - 1) \times m + i - 1 \quad \text{when order} = \text{Nag ColMajor};
\]

\[
\text{colour}(i - 1) \times n + j - 1 \quad \text{when order} = \text{Nag RowMajor}.
\]

On exit: \( \text{COLOUR}(i, j) \), contains the third-order Greek measuring the sensitivity of the second-order Greek \( \Gamma_{ij} \) to change in the time, i.e., \( -\frac{\partial \Gamma_{ij}}{\partial \tau} = -\frac{\partial^3 p_i}{\partial \tau^2 \partial \sigma} \), for \( i = 1, 2, \ldots, m \) and \( j = 1, 2, \ldots, n \).

22: \( \text{zomma}[m \times n] \) – double

Output

Note: where \( \text{ZOMMA}(i, j) \) appears in this document, it refers to the array element

\[
\text{zomma}(j - 1) \times m + i - 1 \quad \text{when order} = \text{Nag ColMajor};
\]

\[
\text{zomma}(i - 1) \times n + j - 1 \quad \text{when order} = \text{Nag RowMajor}.
\]

On exit: \( \text{ZOMMA}(i, j) \), contains the third-order Greek measuring the sensitivity of the second-order Greek \( \Gamma_{ij} \) to change in the volatility of the underlying asset, i.e., \( -\frac{\partial \Gamma_{ij}}{\partial \sigma} = -\frac{\partial^3 p_i}{\partial \sigma^2 \partial \sigma} \), for \( i = 1, 2, \ldots, m \) and \( j = 1, 2, \ldots, n \).
23:  \[\text{vomma}[m \times n]\] – double

Output

Note: where \(\text{VOMMA}(i,j)\) appears in this document, it refers to the array element
\[\text{vomma}[(j-1) \times m + i - 1]\] when \(\text{order} = \text{Nag\_ColMajor}\);
\[\text{vomma}[(i-1) \times n + j - 1]\] when \(\text{order} = \text{Nag\_RowMajor}\).

On exit: \(\text{VOMMA}(i,j)\), contains the second-order Greek measuring the sensitivity of the first-order Greek \(\Delta_{ij}\) to change in the volatility of the underlying asset, i.e.,
\[-\frac{\partial\Delta_{ij}}{\partial\sigma} = -\frac{\partial^2 P_i}{\partial\sigma^2},\]
for \(i = 1,2,\ldots,m\) and \(j = 1,2,\ldots,n\).

24:  fail – NagError *

Input/Output

The NAG error argument (see Section 3.6 in the Essential Introduction).

6  Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.
See Section 3.2.1.2 in the Essential Introduction for further information.

NE_BAD_PARAM

On entry, argument \(\langle\text{value}\rangle\) had an illegal value.

NE_INT

On entry, \(m = \langle\text{value}\rangle\).
Constraint: \(m \geq 1\).

On entry, \(n = \langle\text{value}\rangle\).
Constraint: \(n \geq 1\).

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.
An unexpected error has been triggered by this function. Please contact NAG.
See Section 3.6.6 in the Essential Introduction for further information.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.
See Section 3.6.5 in the Essential Introduction for further information.

NE_REAL

On entry, \(q = \langle\text{value}\rangle\).
Constraint: \(q \geq 0.0\).

On entry, \(r = \langle\text{value}\rangle\).
Constraint: \(r \geq 0.0\).

On entry, \(s = \langle\text{value}\rangle\).
Constraint: \(s \geq \langle\text{value}\rangle\) and \(s \leq \langle\text{value}\rangle\).

On entry, \(\text{sigma} = \langle\text{value}\rangle\).
Constraint: \(\text{sigma} > 0.0\).

NE_REAL_ARRAY

On entry, \(t\langle\text{value}\rangle\) = \(\langle\text{value}\rangle\).
Constraint: \(t[i] \geq \langle\text{value}\rangle\).
On entry, $x[\text{value}] = \langle\text{value}\rangle$.
Constraint: $x[i] \geq \langle\text{value}\rangle$ and $x[i] \leq \langle\text{value}\rangle$.

7 Accuracy

The accuracy of the output is dependent on the accuracy of the cumulative Normal distribution function, $\Phi$. This is evaluated using a rational Chebyshev expansion, chosen so that the maximum relative error in the expansion is of the order of the machine precision (see nag_cumul_normal (s15abc) and nag_erfc (s15adc)). An accuracy close to machine precision can generally be expected.

8 Parallelism and Performance

nag_bsm_greeks (s30abc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users’ Note for your implementation for any additional implementation-specific information.

9 Further Comments

None.

10 Example

This example computes the price of a European put with a time to expiry of 0.7 years, a stock price of 55 and a strike price of 60. The risk-free interest rate is 10% per year and the volatility is 30% per year.

10.1 Program Text

/* nag_bsm_greeks (s30abc) Example Program.
 * * Copyright 2014 Numerical Algorithms Group.
 * * Mark 9, 2009.
 */
#include <stdio.h>
#include <math.h>
#include <string.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
    /* Integer scalar and array declarations */
    Integer exit_status = 0;
    Integer i, j, m, n;
    NagError fail;
    Nag_CallPut putnum;
    /* Double scalar and array declarations */
    double q, r, s, sigma;
    double *charm = 0, *colour = 0, *crho = 0, *delta = 0, *gamma = 0;
    double *p = 0, *rho = 0, *speed = 0, *t = 0, *theta = 0, *vanna = 0;
    double *vega = 0, *vomma = 0, *x = 0, *zomma = 0;
    /* Character scalar and array declarations */
    char put[8+1];
    Nag_OrderType order;

    INIT_FAIL(fail);

    printf("nag_bsm_greeks (s30abc) Example Program Results\n");
    /* Skip heading in data file */
#ifdef _WIN32
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```c
#define CHARM(I, J) charm[(J-1)*m + I-1]
#define COLOUR(I, J) colour[(J-1)*m + I-1]
#define CRHO(I, J) crho[(J-1)*m + I-1]
#define DELTA(I, J) delta[(J-1)*m + I-1]
#define GAMMA(I, J) gamma[(J-1)*m + I-1]
#define P(I, J) p[(J-1)*m + I-1]
#define RHO(I, J) rho[(J-1)*m + I-1]
#define SPEED(I, J) speed[(J-1)*m + I-1]
#define THETA(I, J) theta[(J-1)*m + I-1]
#define VANNA(I, J) vanna[(J-1)*m + I-1]
#define VEGA(I, J) vega[(J-1)*m + I-1]
#define VOMMA(I, J) vomma[(J-1)*m + I-1]
#define ZOMMA(I, J) zomma[(J-1)*m + I-1]
#define GAMMA(I, J) gamma[(J-1)*m + I-1]
#define P(I, J) p[(J-1)*m + I-1]
#define RHO(I, J) rho[(J-1)*m + I-1]
#define SPEED(I, J) speed[(J-1)*m + I-1]
#define THETA(I, J) theta[(J-1)*m + I-1]
#define VANNA(I, J) vanna[(J-1)*m + I-1]
#define VEGA(I, J) vega[(J-1)*m + I-1]
#define VOMMA(I, J) vomma[(J-1)*m + I-1]
#define ZOMMA(I, J) zomma[(J-1)*m + I-1]
```
!(x = NAG_ALLOC(m, double)) ||
!(zomma = NAG_ALLOC(m*n, double)))
{
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
}

/* Read array of strike/exercise prices, X */
for (i = 0; i < m; i++)
#ifdef _WIN32
    scanf_s("%lf ", &x[i]);
#else
    scanf("%lf ", &x[i]);
#endif
#endif _WIN32
    scanf_s("%*[\n ] ");
#else
    scanf("%*[\n ] ");
#endif

/* Read array of times to expiry */
for (i = 0; i < n; i++)
#ifdef _WIN32
    scanf_s("%lf ", &t[i]);
#else
    scanf("%lf ", &t[i]);
#endif
#endif _WIN32
    scanf_s("%*[\n ] ");
#else
    scanf("%*[\n ] ");
#endif

/* nag_bsm_greeks (s30abc) */
/* Black-Scholes-Merton option pricing formula with Greeks */
nag_bsm_greeks(order, putnum, m, n, x, s, t, sigma, r, q, p,
    delta, gamma, vega, theta, rho, crho, vanna, charm,
    speed, colour, zomma, vomma, &fail);
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_bsm_greeks (s30abc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

if (putnum == Nag_Call)
    printf("European Call :

");
else if (putnum == Nag_Put)
    printf("European Put :

");
printf("%s8.4f\n", " Spot = ", s);
printf("%s8.4f\n", " Volatility = ", sigma);
printf("%s8.4f\n", " Rate = ", r);
printf("%s8.4f\n", " Dividend = ", q);
for (j = 1; j <= n; j++)
{
    printf(" Time to Expiry : %8.4f
", t[j-1]);
    printf(" Strike Price Delta Gamma Vega 
" "Theta Rho CRho"
);
    for (i = 1; i <= m; i++)
        printf("%8.4f %8.4f %8.4f %8.4f %8.4f %8.4f %8.4f %8.4f
",
            x[i-1], P(i, j), DELTA(i, j), GAMMA(i, j), VEGA(i, j),
            THETA(i, j), RHO(i, j), CRHO(i, j));
    printf(" Vanna Charm Speed 
" "Colour Zomma Vomma"
);
    for (i = 1; i <= m; i++)
        printf("%26.4f %8.4f %8.4f %8.4f %8.4f
",
            VANNA(i, j), CHARM(i, j), SPEED(i, j),
            COLOUR(i, j), ZOMMA(i, j), VOMMA(i, j));
}

END:
NAG_FREE(charm);
NAG_FREE(colour);
NAG_FREE(crho);
NAG_FREE(delta);
NAG_FREE(gamma);
NAG_FREE(p);
NAG_FREE(rho);
NAG_FREE(speed);
NAG_FREE(t);
NAG_FREE(theta);
NAG_FREE(vanna);
NAG_FREE(vega);
NAG_FREE(vomma);
NAG_FREE(x);
NAG_FREE(zomma);

return exit_status;
}

10.2 Program Data

nag_bsm_greeks (s30abc) Example Program Data

Nag_Call or Nag_Put
55.0 0.3 0.1 0.0 : s, sigma, r, q
1 1 : m, n
60.0 : x(i), i = 1,2,...m
0.7 : t(i), i = 1,2,...n

10.3 Program Results

nag_bsm_greeks (s30abc) Example Program Results

European Put:

<table>
<thead>
<tr>
<th>Strike</th>
<th>Price</th>
<th>Delta</th>
<th>Gamma</th>
<th>Vega</th>
<th>Theta</th>
<th>Rho</th>
<th>Crho</th>
<th>Vanna</th>
<th>Charm</th>
<th>Speed</th>
<th>Colour</th>
<th>Zomma</th>
<th>Vomma</th>
</tr>
</thead>
<tbody>
<tr>
<td>60.0000</td>
<td>6.0245</td>
<td>-0.4770</td>
<td>0.0289</td>
<td>18.3273</td>
<td>-0.7014</td>
<td>-22.5811</td>
<td>-18.3639</td>
<td>0.2566</td>
<td>-0.2137</td>
<td>-0.0006</td>
<td>0.0215</td>
<td>-0.0972</td>
<td>-0.6816</td>
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</tr>
</tbody>
</table>

Spot = 55.0000
Volatility = 0.3000
Rate = 0.1000
Dividend = 0.0000

Time to Expiry = 0.7000