1 Purpose

nag_general_elliptic_integral_f (s21dac) returns the value of the general elliptic integral of the second kind \( F(z, k', a, b) \) for a complex argument \( z \).

2 Specification

```c
#include <nag.h>
#include <nags.h>
Complex nag_general_elliptic_integral_f (Complex z, double akp, double a, double b, NagError *fail)
```

3 Description

nag_general_elliptic_integral_f (s21dac) evaluates an approximation to the general elliptic integral of the second kind \( F(z, k', a, b) \) given by

\[
F(z, k', a, b) = \int_0^z \frac{a + b\zeta^2}{(1 + \zeta^2)(1 + k'^2\zeta^2)} d\zeta,
\]

where \( a \) and \( b \) are real arguments, \( z \) is a complex argument whose real part is non-negative and \( k' \) is a real argument (the complementary modulus). The evaluation of \( F \) is based on the Gauss transformation. Further details, in particular for the conformal mapping provided by \( F \), can be found in Bulirsch (1960).

Special values include

\[
F(z, k', 1, 1) = \int_0^z \frac{d\zeta}{\sqrt{(1 + \zeta^2)(1 + k'^2\zeta^2)}},
\]

or \( F_1(z, k') \) (the elliptic integral of the first kind) and

\[
F\left(z, k', 1, k'^2\right) = \int_0^z \frac{\sqrt{1 + k'^2\zeta^2}}{(1 + \zeta^2)\sqrt{1 + k'^2\zeta^2}} d\zeta,
\]

or \( F_2(z, k') \) (the elliptic integral of the second kind). Note that the values of \( F_1(z, k') \) and \( F_2(z, k') \) are equal to \( \tan^{-1}(z) \) in the trivial case \( k' = 1 \).

nag_general_elliptic_integral_f (s21dac) is derived from an Algol 60 procedure given by Bulirsch (1960). Constraints are placed on the values of \( z \) and \( k' \) in order to avoid the possibility of machine overflow.

4 References


5 Arguments

1: \( z \) – Complex

\( \text{Input} \)

\( \text{On entry:} \) the argument \( z \) of the function.
Constraints:
0.0 \leq z.re \leq \lambda;
\text{abs}(z.im) \leq \lambda, \text{ where } \lambda^6 = 1/nag\_real\_safe\_small\_number.

2: akp – double  \hspace{1cm} Input
On entry: the argument \(k'\) of the function.
Constraint: \text{abs}(akp) \leq \lambda.

3: a – double  \hspace{1cm} Input
On entry: the argument \(a\) of the function.

4: b – double  \hspace{1cm} Input
On entry: the argument \(b\) of the function.

5: fail – NagError*  \hspace{1cm} Input/Output
The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_ALLOC_FAIL
Dynamic memory allocation failed.
See Section 3.2.1.2 in the Essential Introduction for further information.

NE_COMPLEX
On entry, \(|z.im|\) is too large: \(|z.im| = \langle value\rangle. It must not exceed \langle value\rangle.
On entry, \(z.re < 0.0: z.re = \langle value\rangle.
On entry, \(z.re\) is too large: \(z.re = \langle value\rangle. It must not exceed \langle value\rangle.

NE_INTERNAL_ERROR
An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.
An unexpected error has been triggered by this function. Please contact NAG.
See Section 3.6.6 in the Essential Introduction for further information.

NE_NO_LICENCE
Your licence key may have expired or may not have been installed correctly.
See Section 3.6.5 in the Essential Introduction for further information.

NE_REAL
On entry, \(|akp|\) is too large: \(|akp| = \langle value\rangle. It must not exceed \langle value\rangle.

NE_S21_CONV
The iterative procedure used to evaluate the integral has failed to converge.

7 Accuracy
In principle the function is capable of achieving full relative precision in the computed values. However, the accuracy obtainable in practice depends on the accuracy of the standard elementary functions such as atan2 and log.
8 Parallelism and Performance

Not applicable.

9 Further Comments

None.

10 Example

This example evaluates the elliptic integral of the first kind \( F_1(z, k') \) given by

\[
F_1(z, k') = \int_0^z \frac{d\zeta}{\sqrt{(1 + \zeta^2)(1 + k'^2\zeta^2)}}
\]

where \( z = 1.2 + 3.7i \) and \( k' = 0.5 \), and prints the results.

10.1 Program Text

/* nag_general_elliptic_integral_f (s21dac) Example Program. *
* * Copyright 2014 Numerical Algorithms Group. *
* * NAG C Library *
* * Mark 6, 2000. *
*/
#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
    Complex y, z;
    Integer exit_status = 0;
    NagError fail;
    double a, akp, b;

    INIT_FAIL(fail);

    /* Skip heading in data file */
    #ifdef _WIN32
        scanf_s("%*[^
] ");
    #else
        scanf("%*[^
] ");
    #endif

    printf("nag_general_elliptic_integral_f (s21dac) Example Program 
"Results\n");
    printf(" z akp a b y\n");
    #ifdef _WIN32
        while (scanf_s(" (%lf,%lf) %lf %lf %lf%*[\n] ",
            &z.re, &z.im, &akp, &a, &b) != EOF)
    #else
        while (scanf(" (%lf,%lf) %lf %lf %lf%*[\n] ",
            &z.re, &z.im, &akp, &a, &b) != EOF)
    #endif
    { /* nag_general_elliptic_integral_f (s21dac).
        * Elliptic integrals of the second kind with complex 
        * arguments
        */
        y = nag_general_elliptic_integral_f(z, akp, a, b, &fail);
    }
    return 0;
}
if (fail.code == NE_NOERROR)
    printf("(%4.1f, %4.1f) %7.1f %7.1f %7.1f (%13.4e, %13.4e)\n",
            z.re, z.im, akp, a, b, y.re, y.im);
else
{
    printf("Error from nag_general_elliptic_integral_f (s21dac)."
            "\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
END:
return exit_status;
}

10.2 Program Data

nag_general_elliptic_integral_f (s21dac) Example Program Data
(1.2, 3.7) 0.5 1.0 1.0
(9.2, -3.4) 0.8 0.4 2.7 : Values of z, akp, a and b

10.3 Program Results

nag_general_elliptic_integral_f (s21dac) Example Program Results
z      akp      a      b      y
( 1.2, 3.7) 0.5 1.0 1.0 ( 1.9713e+00, 5.0538e-01)
( 9.2, -3.4) 0.8 0.4 2.7 ( 2.5042e+00, -1.1709e-01)