NAG Library Function Document

nag_kelvin_bei_vector (s19apc)

1 Purpose
nag_kelvin_bei_vector (s19apc) returns an array of values for the Kelvin function bei \( x \).

2 Specification
#include <nag.h>
#include <nags.h>
void nag_kelvin_bei_vector (Integer n, const double x[], double f[],
   Integer ivalid[], NagError *fail)

3 Description
nag_kelvin_bei_vector (s19apc) evaluates an approximation to the Kelvin function bei \( x \) for an array of arguments \( x_i \), for \( i = 1, 2, \ldots, n \).

Note: bei(\(-x\)) = bei \( x \), so the approximation need only consider \( x \geq 0.0 \).

The function is based on several Chebyshev expansions:
For \( 0 \leq x \leq 5 \),
\[ \text{bei} x = \frac{x^2}{4} \sum_{r=0} a_r T_r(t) \quad \text{with } t = 2 \left( \frac{x}{5} \right)^4 - 1; \]
For \( x > 5 \),
\[ \text{bei} x = e^{x/\sqrt{2}} \left( \left( 1 + \frac{1}{x} a(t) \right) \sin \alpha - \frac{1}{x} b(t) \cos \alpha \right) \]
\[ + e^{x/\sqrt{2}} \left( \left( 1 + \frac{1}{x} c(t) \right) \cos \beta - \frac{1}{x} d(t) \sin \beta \right) \]
where \( \alpha = \frac{x}{\sqrt{2}} - \frac{\pi}{8}, \beta = \frac{x}{\sqrt{2}} + \frac{\pi}{8} \),
and \( a(t), b(t), c(t), \) and \( d(t) \) are expansions in the variable \( t = \frac{10}{x} - 1 \).

When \( x \) is sufficiently close to zero, the result is computed as bei \( x = \frac{x^2}{4} \). If this result would underflow, the result returned is bei \( x = 0.0 \).

For large \( x \), there is a danger of the result being totally inaccurate, as the error amplification factor grows in an essentially exponential manner; therefore the function must fail.

4 References
5 Arguments

1:  \textbf{n} – Integer \hspace{1cm} \textit{Input} \\
On entry: \( n \), the number of points. \\
\textit{Constraint:} \( n \geq 0 \).

2:  \( x[n] \) – const double \hspace{1cm} \textit{Input} \\
On entry: the argument \( x_i \) of the function, for \( i = 1, 2, \ldots, n \).

3:  \( f[n] \) – double \hspace{1cm} \textit{Output} \\
On exit: \( \text{bei} x_i \), the function values.

4:  \textbf{invalid}[n] – Integer \hspace{1cm} \textit{Output} \\
On exit: \textbf{invalid}[i - 1] \text{ contains the error code for } x_i, \text{ for } i = 1, 2, \ldots, n. \\
\textbf{invalid}[i - 1] = 0 \\
No error. \\
\textbf{invalid}[i - 1] = 1 \\
\text{abs}(x_i) \text{ is too large for an accurate result to be returned. \( f[i - 1] \) contains zero. The threshold value is the same as for fail.code = NE_REAL_ARG_GT in nag_kelvin_bei (s19abc), as defined in the Users’ Note for your implementation.}

5:  \textbf{fail} – NagError * \hspace{1cm} \textit{Input/Output} \\
The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

**NE_ALLOC_FAIL**
Dynamic memory allocation failed. 
See Section 3.2.1.2 in the Essential Introduction for further information.

**NE_BAD_PARAM**
On entry, argument \( \langle \text{value} \rangle \) had an illegal value.

**NE_INT**
On entry, \( n = \langle \text{value} \rangle \). 
\textit{Constraint:} \( n \geq 0 \).

**NE_INTERNAL_ERROR**
An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance. 
An unexpected error has been triggered by this function. Please contact NAG. 
See Section 3.6.6 in the Essential Introduction for further information.

**NE_NO_LICENCE**
Your licence key may have expired or may not have been installed correctly. 
See Section 3.6.5 in the Essential Introduction for further information.

**NW_IVALID**
On entry, at least one value of \( x \) was invalid. 
Check \textbf{invalid} for more information.
7 Accuracy

Since the function is oscillatory, the absolute error rather than the relative error is important. Let \( E \) be the absolute error in the function, and \( \delta \) be the relative error in the argument. If \( \delta \) is somewhat larger than the machine precision, then we have:

\[
E \approx \frac{x}{\sqrt{2}} \left( -\text{ber}_1 x + \text{bei}_1 x \right) \delta
\]

(provided \( E \) is within machine bounds).

For small \( x \) the error amplification is insignificant and thus the absolute error is effectively bounded by the machine precision.

For medium and large \( x \), the error behaviour is oscillatory and its amplitude grows like \( \sqrt{x e^{x/\sqrt{2}}} \).

Therefore it is impossible to calculate the functions with any accuracy when \( \sqrt{x e^{x/\sqrt{2}}} > \frac{\sqrt{2\pi}}{\delta} \). Note that this value of \( x \) is much smaller than the minimum value of \( x \) for which the function overflows.

8 Parallelism and Performance

Not applicable.

9 Further Comments

None.

10 Example

This example reads values of \( x \) from a file, evaluates the function at each value of \( x_i \) and prints the results.

10.1 Program Text

/* nag_kelvin_bei_vector (s19apc) Example Program. */
/* Copyright 2014 Numerical Algorithms Group. */
/* Mark 23, 2011. */
/*
#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
    Integer exit_status = 0;
    Integer i, n;
    double *f = 0, *x = 0;
    Integer *ivalid = 0;
    NagError fail;

    INIT_FAIL(fail);

    /* Skip heading in data file */
    #ifdef _WIN32
        scanf_s("%*\n");
    #else
        scanf("%*\n");
    #endif

    printf("nag_kelvin_bei_vector (s19apc) Example Program Results\n");

    ...
printf("\n");
printf(" x f ivalid\n");
printf("\n");

#ifdef _WIN32
scanf_s("%"NAG_IFMT", &n);
#else
scanf("%"NAG_IFMT", &n);
#endif
#ifdef _WIN32
scanf_s("%*[\n]");
#else
scanf("%*[\n]");
#endif

/* Allocate memory */
if (!(x = NAG_ALLOC(n, double)) ||
! (f = NAG_ALLOC(n, double)) ||
! (ivalid = NAG_ALLOC(n, Integer)))
{
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
}

for (i=0; i<n; i++)
#ifdef _WIN32
    scanf_s("%lf", &x[i]);
#else
    scanf("%lf", &x[i]);
#endif
#ifdef _WIN32
    scanf_s("%*[\n]");
#else
    scanf("%*[\n]");
#endif

/* nag_kelvin_bei_vector (s19apc).
 * Kelvin Function bei x */
舴g_kelvin_bei_vector(n, x, f, ivalid, &fail);
if (fail.code!=NE_NOERROR && fail.code!=NW_IVALID)
{
    printf("Error from nag_kelvin_bei_vector (s19apc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

for (i=0; i<n; i++)
    printf(" %11.3e %11.3e %4"NAG_IFMT"\n", x[i], f[i], ivalid[i]);

END:
NAG_FREE(f);
NAG_FREE(x);
NAG_FREE(ivalid);
return exit_status;
}

10.2 Program Data

nag_kelvin_bei_vector (s19apc) Example Program Data

7
0.1 1.0 2.5 5.0 10.0 15.0 -1.0
### 10.3 Program Results

nag_kelvin_bei_vector (s19apc) Example Program Results

<table>
<thead>
<tr>
<th>x</th>
<th>f</th>
<th>ivalid</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.000e-01</td>
<td>2.500e-03</td>
<td>0</td>
</tr>
<tr>
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<td>2.496e-01</td>
<td>0</td>
</tr>
<tr>
<td>2.500e+00</td>
<td>1.457e+00</td>
<td>0</td>
</tr>
<tr>
<td>5.000e+00</td>
<td>1.160e-01</td>
<td>0</td>
</tr>
<tr>
<td>1.000e+01</td>
<td>5.637e+01</td>
<td>0</td>
</tr>
<tr>
<td>1.500e+01</td>
<td>-2.953e+03</td>
<td>0</td>
</tr>
<tr>
<td>-1.000e+00</td>
<td>2.496e-01</td>
<td>0</td>
</tr>
</tbody>
</table>