NAG Library Function Document

nag_bessel_k_nu_scaled (s18edc)

1 Purpose
nag_bessel_k_nu_scaled (s18edc) returns the value of the scaled modified Bessel function $e^{x}K_{\nu/4}(x)$ for real $x > 0$.

2 Specification
#include <nag.h>
#include <nags.h>
double nag_bessel_k_nu_scaled (double x, Integer nu, NagError *fail)

3 Description
nag_bessel_k_nu_scaled (s18edc) evaluates an approximation to the scaled modified Bessel function of the second kind $e^{x}K_{\nu/4}(x)$, where the order $\nu = -3, -2, -1, 1, 2$ or $3$ and $x$ is real and positive. For negative orders the formula

$$K_{-\nu/4}(x) = K_{\nu/4}(x)$$

is used.

4 References

5 Arguments
1:  x – double  
   On entry: the argument $x$ of the function.  
   Constraint: $x > 0.0$.

2:  nu – Integer  
   On entry: the argument $\nu$ of the function.  
   Constraint: $1 \leq \text{abs}(\text{nu}) \leq 3$.

3:  fail – NagError *  
   The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_INT
   On entry, nu = (value).  
   Constraint: $1 \leq \text{abs}(\text{nu}) \leq 3$.
NE_INTERNAL_ERROR
An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

NE_OVERFLOW_LIKELEY
The evaluation has been abandoned due to the likelihood of overflow. The result is returned as zero.

NE_REAL
On entry, \( x = \langle \text{value} \rangle \).
Constraint: \( x > 0.0 \).

NE_TERMINATION_FAILURE
The evaluation has been abandoned due to failure to satisfy the termination condition. The result is returned as zero.

NE_TOTAL_PRECISION_LOSS
The evaluation has been abandoned due to total loss of precision. The result is returned as zero.

NW_SOME_PRECISION_LOSS
The evaluation has been completed but some precision has been lost.

7 Accuracy
All constants in the underlying function are specified to approximately 18 digits of precision. If \( t \) denotes the number of digits of precision in the floating-point arithmetic being used, then clearly the maximum number of correct digits in the results obtained is limited by \( p = \min(t, 18) \). Because of errors in argument reduction when computing elementary function inside the underlying function, the actual number of correct digits is limited, in general, by \( p - s \), where \( s \approx \max(1, |\log_{10} x|) \) represents the number of digits lost due to the argument reduction. Thus the larger the value of \( x \), the less the precision in the result.

8 Parallelism and Performance
Not applicable.

9 Further Comments
None.

10 Example
The example program reads values of the arguments \( x \) and \( \nu \) from a file, evaluates the function and prints the results.

10.1 Program Text
/* nag_bessel_k_nu_scaled (s18edc) Example Program. */
/* Copyright 2014 Numerical Algorithms Group. */
/* NAG C Library */
/* Mark 6, 2000. */
```c
#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
    Integer exit_status = 0, nu;
    NagError fail;
    double x, y;

    INIT_FAIL(fail);

    /* Skip heading in data file */
    #ifdef _WIN32
        scanf_s("%*[\n]");
    #else
        scanf("%*[\n]");
    #endif

    printf("nag_bessel_k_nu_scaled (s18edc) Example Program Results\n");
    printf("x  nu  y\n");
    #ifdef _WIN32
        while (scanf_s("%lf %f*[\n]", &x, &nu) != EOF)
    #else
        while (scanf("%lf %f*[\n]", &x, &nu) != EOF)
    #endif
    {
        /* nag_bessel_k_nu_scaled (s18edc).
           * Scaled modified Bessel function exp(x) K_(nu/4)(x)
        */
        y = nag_bessel_k_nu_scaled(x, nu, &fail);
        if (fail.code != NE_NOERROR)
        {
            printf("Error from nag_bessel_k_nu_scaled (s18edc).\ns\n", fail.message);
            exit_status = 1;
            goto END;
        }
        printf("%4.1f %6f %13.4e\n", x, nu, y);
    }

    END:
    return exit_status;
}
```

### 10.2 Program Data

nag_bessel_k_nu_scaled (s18edc) Example Program Data

<table>
<thead>
<tr>
<th>x</th>
<th>ν</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.9</td>
<td>-3</td>
<td>6.5781e-01</td>
</tr>
<tr>
<td>1.4</td>
<td>-2</td>
<td>1.0592e+00</td>
</tr>
<tr>
<td>8.2</td>
<td>-1</td>
<td>4.3297e-01</td>
</tr>
<tr>
<td>6.7</td>
<td>1</td>
<td>4.7791e-01</td>
</tr>
<tr>
<td>0.5</td>
<td>2</td>
<td>1.7725e+00</td>
</tr>
<tr>
<td>2.3</td>
<td>3</td>
<td>8.7497e-01</td>
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</tbody>
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### 10.3 Program Results

nag_bessel_k_nu_scaled (s18edc) Example Program Results

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